Basic Probability

1. The table below shows the number of left and right handed tennis players in a sample of 50 males and females.

<table>
<thead>
<tr>
<th></th>
<th>Left handed</th>
<th>Right handed</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>3</td>
<td>29</td>
<td>32</td>
</tr>
<tr>
<td>Female</td>
<td>2</td>
<td>16</td>
<td>18</td>
</tr>
<tr>
<td>Total</td>
<td>5</td>
<td>45</td>
<td>50</td>
</tr>
</tbody>
</table>

If a tennis player was selected at random from the group, find the probability that the player is

(a) male and left handed;  
(b) right handed;  
(c) right handed, given that the player selected is female.  

\[
\begin{align*}
\text{(a) } & \frac{3}{50} \text{ or } 6\% \text{ or } 0.06 \\
\text{(b) } & \frac{45}{50} \text{ or } \frac{9}{10} \text{ or } 90\% \text{ or } 0.9 \\
\text{(c) } & \frac{16}{18} \text{ or } \frac{8}{9} \text{ or } 0.889 \text{ (3 s.f.)}
\end{align*}
\]

2. It is known that 5% of all AA batteries made by Power Manufacturers are defective. AA batteries are sold in packs of 4. Find the probability that a pack of 4 has

(a) exactly two defective batteries;  
(b) at least one defective battery.  

\[
\begin{align*}
\text{(a) } & p(\text{two defective}) = 6 \times 0.05^2 \times 0.95^2 = 0.0135375 = 0.0135 \text{ (3 s.f.)} \\
\text{(b) } & p(\text{at least one defective}) = 1 - 0.95^4 = 0.18549375... = 0.0185 \text{ (3 s.f.)}
\end{align*}
\]

3. A survey of 400 people is carried out by a market research organization in two different cities, Buenos Aires and Montevideo. The people are asked which brand of cereal they prefer out of Chocos, Zucos or Fruti. The table below summarizes their responses.

<table>
<thead>
<tr>
<th></th>
<th>Chocos</th>
<th>Zucos</th>
<th>Fruti</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buenos Aires</td>
<td>43</td>
<td>85</td>
<td>62</td>
<td>190</td>
</tr>
<tr>
<td>Montevideo</td>
<td>57</td>
<td>35</td>
<td>118</td>
<td>210</td>
</tr>
<tr>
<td>Total</td>
<td>100</td>
<td>120</td>
<td>180</td>
<td>400</td>
</tr>
</tbody>
</table>

(a) One person is chosen at random from those surveyed. Find the probability that this person

(i) does not prefer Zucos;  
(ii) prefers Chocos, given that they live in Montevideo.  
(b) Two people are chosen at random from those surveyed. Find the probability that they both prefer Fruti.  

\[
\begin{align*}
\text{(a) (i) } & \frac{280}{400} \text{ (0.7, 70\% or equivalent )} \\
\text{(ii) } & \frac{57}{210} \left( \frac{19}{70}, 0.271, 27.1\% \right) \\
\text{(b) } & \frac{180}{400} \times \frac{179}{399} = \frac{537}{2660} = 0.202
\end{align*}
\]

4. The local Football Association consists of ten teams. Team A has a 40% chance of winning any game against a higher-ranked team, and a 75% chance of winning any game against a lower-ranked team. If A is currently in fourth position, find the probability that A wins its next game.  

\[
\begin{align*}
\text{Required probability } = \frac{3}{9} \times \frac{2}{5} + \frac{6}{9} \times \frac{3}{4} = \frac{19}{30}
\end{align*}
\]
5. In a group of fifteen students, three names begin with the letter B and four begin with a G. The remaining eight names begin with A, C, D, E, F, H, I and J respectively. The 15 names are placed in a box. The box is shaken and two names are drawn out. Find the probability that

(a) both names begin with any letter except G or B;
(b) both names begin with the same letter;
(c) both names begin with the letter H.

\[ \left( \frac{8}{15} \times \frac{7}{14} \right) = \frac{56}{210} = \frac{4}{15} \quad (0.267) \]

\[ \left( \frac{4}{15} \times \frac{3}{14} \right) + \left( \frac{3}{15} \times \frac{2}{14} \right) = \frac{18}{210} \quad or \quad \frac{3}{55} \quad (0.0857) \]

(c) The probability is 0. (Allow answer “impossible” or equivalent.)

6. When John throws a stone at a target, the probability that he hits the target is 0.4. He throws a stone 6 times.

(a) Find the probability that he hits the target exactly 4 times.

(b) Find the probability that he hits the target for the first time on his third throw.

(a) Probability = \( \binom{6}{4} \times (0.4)^4 \times (0.6)^2 = 0.138 \quad \text{(accept} \frac{432}{3125} \text{or} 0.13824) \)

(b) Probability = \( (0.6)^2 \times 0.4 = 0.144 \quad \text{(or} \frac{18}{125}) \)

7. In a group of 20 students, there are 12 girls and 8 boys. Two of the boys and three of the girls wear red shirt. What is the probability that the person chosen randomly from the group is either a boy or someone who wears a red shirt?

\[ P(\text{boy}) = \frac{8}{20} \quad P(\text{red shirt}) = \frac{5}{20} \quad P(\text{boy} \cap \text{red shirt}) = \frac{2}{20} \]

\[ P(\text{boy} \cup \text{red shirt}) = P(\text{boy}) + P(\text{red shirt}) - P(\text{boy} \cap \text{red shirt}) = \frac{8}{20} + \frac{5}{20} - \frac{2}{20} = \frac{11}{20} \]

or

\[ P(\text{boy} \cup \text{red shirt}) = \frac{8}{20} \left( \frac{2}{8} + \frac{6}{8} \right) + \frac{12}{20} \left( \frac{3}{12} \right) = \frac{11}{20} \]
8. In a promotion to try to increase the sales of a particular brand of breakfast cereal, a picture of a soccer player is put in each packet. There are ten different pictures available. Each picture is equally likely to be found in any packet of breakfast cereal.

Charlotte buys four packets of breakfast cereal.

(a) Find the probability that the four pictures in these packets are all different.

(b) Of the ten players whose pictures are in the packets, her favourites are Alan and Bob. Find the probability that she finds at least one picture of a favourite player in these four packets.

\[ P(\text{four different}) = 1 \times \frac{9}{10} \times \frac{8}{10} \times \frac{7}{10} = \frac{504}{1000} = 0.504 \]

(b) In any packet the probability of not getting pictures of Alan or Bob = \( \frac{8}{10} \)

In four packets the probability of not getting pictures of Alan or Bob is \( \left( \frac{8}{10} \right)^4 \)

Required probability is \( 1 - \left( \frac{8}{10} \right)^4 = 0.590 \)

9. A man visits his local supermarket twice in a week. The probability that he pays by credit card is 0.4 and the probability that he pays with cash is 0.6. Find the probability that

(a) he pays cash on both visits

(b) he pays cash on the first visit and by credit card on the second visit.

\[ P(\text{both cash}) = 0.6 \cdot 0.6 = 0.36 \]

\[ P(\text{cash first, card second}) = 0.6 \cdot 0.4 = 0.24 \]

10. A student is practicing her goal scoring for soccer. The probability that the ball hits the net on any particular attempt is 0.7 and she does not improve with practice.

(a) Find how many balls should be kicked so that the probability that she hits the net at least once is greater than 0.995.

(b) Find how many balls should be kicked so that the probability that she does not hit is less than 0.001.
11. A box contains 10 coloured light bulbs, 5 green, 3 red and 2 yellow. One light bulb is selected at random and put into the light fitting of room A.
   (a) What is the probability that the light bulb selected is
      (i) green?
      (ii) not green?

   A second light bulb is selected at random and put into the light fitting in room B.
   (b) What is the probability that
      (i) the second light bulb is green given the first light bulb was green?
      (ii) both light bulbs are not green?
      (iii) one room has a green light bulb and the other room does not have a green light bulb?

   A third light bulb is selected at random and put in the light fitting of room C.
   (c) What is the probability that
      (i) all three rooms have green light bulbs?
      (ii) only one room has a green light bulb?
      (iii) at least one room has a green light bulb?

   (a) (i) \( p(\text{green}) = \frac{5}{10} \)  
                  (ii) \( p(\text{not green}) = \frac{5}{10} \)

   (b) (i) \( p(G \mid G) = \frac{4}{9} \)  or 0.444 (3 s.f.)

   (ii) \( p(\text{not green then not green}) = \frac{5}{10} \times \frac{4}{9} = \frac{20}{90} = \frac{2}{9} \)  or 0.222 (3 s.f.)

   (iii) \( p(\text{one green and one not green}) = \frac{5}{10} \times \frac{5}{9} + \frac{5}{10} \times \frac{5}{9} = \frac{50}{90} \)  or \( \frac{5}{9} \)  or 0.556
(c) (i) \[ p(3 \text{ green}) = \frac{5}{10} \times \frac{4}{9} \times \frac{3}{8} = \frac{60}{720} = \frac{1}{12} \text{ or } 0.0833 \text{ (3 s.f.)} \]

(ii) \[ p(\text{only one green}) = 3 \times \frac{5}{10} \times \frac{5}{9} \times \frac{4}{8} = \frac{300}{720} = \frac{5}{24} \text{ or } 0.208 \text{ (3 s.f.)} \]

(iii) \[ p(\text{at least one green}) = 1 - p(\text{no green}) = 1 - \frac{5}{10} \times \frac{4}{9} \times \frac{3}{8} = 1 - \frac{60}{720} = \frac{11}{12} \text{ or } 0.917 \text{ (3 s.f.)} \]

12. On a certain game show, contestants spin a wheel to win a prize, as shown in the diagram. The larger angles are 40° (the shaded sectors), and the smaller angles are 20°.

Find the probability that a contestant
(a) will not win a prize;
(b) will win a holiday in Greece (GH);
(c) will win a washer/dryer (WD), given that he knows that he has won a prize;
(d) will win a holiday in Greece or a washer/dryer.

![Diagram of the game show wheel]

\[
\begin{align*}
\text{(a)} & \quad \frac{40 \times 6}{360} = \frac{240}{360} = \frac{2}{3} \text{ or } 0.667 \text{ (3 s.f.)} \\
\text{(b)} & \quad \frac{2 \times 20}{360} = \frac{40}{360} = \frac{1}{9} \text{ or } 0.111 \text{ (3 s.f.)} \\
\text{(c)} & \quad \frac{3 \times 20}{120} = \frac{60}{120} = \frac{1}{2} \text{ or } 0.5 \\
\text{(d)} & \quad \frac{100}{360} = \frac{5}{18} \text{ or } 0.278 \text{ (3 s.f.)}
\end{align*}
\]

13. Of a group of five students, two will be selected to visit the United Nations. The five students are John, Maria, Raul, Henri and Susan.
(a) With the aid of a tree diagram or a table of outcomes, find the number of different possible combinations of students that could go to the United Nations.

(b) Find the probability that both Maria and Susan will go on the trip.
14. A group of 25 females were asked how many children they each had. The results are shown in the histogram below.

(a) Show that the mean number of children per female is 1.4.

(b) Show clearly that the standard deviation for this data is approximately 1.06.

(c) Another group of 25 females was surveyed and it was found that the mean number of children per female was 2.4 and the standard deviation was 2. Use the results from parts (a) and (b) to describe the differences between the number of children the two groups of females have.

(d) A female is selected at random from the first group. What is the probability that she has more than two children?

(e) Two females are selected at random from the first group. What is the probability that
   (i) both females have more than two children?
   (ii) only one of the females has more than two children?
   (iii) the second female selected has two children given that the first female selected had no children?

\[
\text{(a) Mean } = \frac{5 \times 0 + 10 \times 1 + 6 \times 2 + 3 \times 3 + 1 \times 4}{25} = 1.4
\]

\[
\text{(b) } \sum f(x - \bar{x})^2 = 5(0 - 1.4)^2 + 10(1 - 1.4)^2 + 6(2 - 1.4)^2 + 3(3 - 1.4)^2 + 1(4 - 1.4)^2 = 28
\]

\[
\text{S.D. } = \sqrt{\frac{28}{25}} = 1.06
\]

(c) each acceptable reason, e.g.
   Group 2 has more children in total.
   Group 2 has a larger number of children per female.
   Group 2 generally have larger families.
(d) \( P(> 2 \text{ children}) = \frac{3+1}{25} = \frac{4}{25} \)

(e) (i) \( P(\text{both females have } > 2 \text{ children}) = \frac{4}{25} \times \frac{3}{24} = \frac{12}{600} \) or \( \frac{1}{50} \) or 0.02

(ii) \( P(\text{only 1 female has } > 2 \text{ children}) = 2 \times \frac{4}{25} \times \frac{21}{24} = \frac{168}{600} \) or \( \frac{21}{75} \) or 0.28

(iii) \( P(\text{second has 2 children } | \text{ first has 0}) = \frac{6}{24} \) or \( \frac{1}{4} \) or 0.25

15. Nene and Deka both play netball. The probability that Nene will score a goal on her first attempt is 0.75. The probability that Deka will score a goal on her first attempt is 0.82. Calculate the probability that

(a) Nene and Deka will both score a goal on their first attempts;

(b) neither Nene nor Deka will score a goal on their first attempts.

(a) \( 0.75 \times 0.82 = 0.615 \) (accept 61.5% or \( \frac{123}{200} \))

(b) \( 0.25 \times 0.18 = 0.045 \) (accept 4.5% or \( \frac{9}{200} \))

16. In a club with 60 members, everyone attends either on Tuesday for Drama (D) or on Thursday for Sports (S) or on both days for Drama and Sports. One week it is found that 48 members attend for Drama and 44 members attend for Sports and \( x \) members attend for both Drama and Sports.

(a) (i) Draw and label fully a Venn diagram to illustrate this information.

(ii) Find the number of members who attend for both Drama and Sports.

(iii) Describe, in words, the set represented by \( (D \cap S)' \).

(iv) What is the probability that a member selected at random attends for Drama only or Sports only?

The club has 28 female members, 8 of whom attend for both Drama and Sports.

(b) What is the probability that a member of the club selected at random

(i) is female and attends for Drama only or Sports only?

(ii) is male and attends for both Drama and Sports?

\[ 48 - x + x + 44 - x = 60 \) (or equivalent), allow \( x \) from (i) \( \Rightarrow x = 32 \)

(iii) The set of members who \textbf{did not} attend for both Drama and Sports (or equivalent)
17. The sets $A$, $B$ and $C$ are subsets of $U$. They are defined as follows:

$U = \{\text{positive integers less than 16}\}$

$A = \{\text{prime numbers}\}$

$B = \{\text{factors of 36}\}$

$C = \{\text{multiples of 4}\}$

(a)
(i) $A$;
(ii) $B$;
(iii) $C$;
(iv) $A \cap B \cap C$.

(b)
(i) Draw a Venn diagram showing the relationship between the sets $U$, $A$, $B$ and $C$.
(ii) Write the elements of sets $U$, $A$, $B$ and $C$ in the appropriate places on the Venn diagram.

(c)
From the Venn diagram, list the elements of each of the following

(i) $A \cap (B \cup C)$;
(ii) $(A \cap B)'$;
(iii) $(A \cap B)' \cap C$.

(d)
Find the probability that a number chosen at random from the universal set $U$ will be

(i) a prime number;
(ii) a prime number, but not a factor of 36;
(iii) a factor of 36 or a multiple of 4, but not a prime number;
(iv) a prime number, given that it is a factor of 36.

(a)
(i) $A = \{2, 3, 5, 7, 11, 13\}$
(ii) $B = \{1, 2, 3, 4, 6, 9, 12\}$
(iii) $C = \{4, 8, 12\}$
(iv) $A \cap B \cap C = \emptyset$

(b)
(i)(ii)

(c)
(i) $A \cap (B \cup C) = \{2, 3\}$
(ii) $(A \cap B)' = \{1, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15\}$
(iii) $(A \cap B)' \cap C = \{4, 8, 12\}$

(d)
(i) $\frac{6}{15} = \frac{2}{5}$ or 0.4 or 40%
(ii) $\frac{4}{15}$ or 0.267 or 26.7% (3 s.f)
(iii) $\frac{6}{15}$ or $\frac{2}{5}$ or 0.4 or 40%
(iv) $\frac{2}{7}$ or 0.286 or 28.6% (3 s.f.)
18.  Let $F$ be the set of all families that have exactly 2 children.
(a)  Assuming $P(\text{boy}) = P(\text{girl})$, draw tree diagram, for families with 2 children.
(b)  What is the probability that a family chosen at random from $F$ has exactly
   (i)  2 boys?
   (ii) 2 boys, if it is known that the first child is a boy?
   (iii) 2 boys, if it is known that there is a boy in the family?

   \[
   \begin{array}{c}
   \text{Boy} \\
   \frac{1}{2} \\
   \text{Boy} \\
   \frac{1}{2} \\
   \text{Girl} \\
   \frac{1}{2} \\
   \text{Boy} \\
   \frac{1}{2} \\
   \text{Girl} \\
   \frac{1}{2}
   \end{array}
   \]

   (b)  \(P(2 \text{ boys}) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}\)
   (ii)  \(P(2 \text{ boys} | \text{first child is boy}) = \frac{\frac{1}{4}}{\frac{1}{2}} = \frac{1}{2}\)
   (iii) \(P(2 \text{ boys} | \text{boy in family}) = \frac{1}{\frac{3}{4}} = \frac{4}{3}\)

19.  The Venn diagram below shows the number of students studying Science ($S$), Mathematics ($M$) and History ($H$) out of a group of 20 college students. Some of the students do not study any of these subjects, 8 study Science, 10 study Mathematics and 9 study History.

\[
\begin{array}{c}
U \\
S \\
1 \\
A \\
4 \\
M \\
1 \\
H \\
3 \\
A \\
2 \\
H \\
3
\end{array}
\]

(a)  (i)  How many students belong to the region labelled $A$?
   (ii)  Describe in words the region labelled $A$.
   (iii)  How many students do not study any of the three subjects?
(b)  Draw a sketch of the Venn diagram above and shade the region which represents $S' \cap H$.
(c)  Calculate $n(S \cup H)$.

This group of students is to compete in an annual quiz evening which tests knowledge of Mathematics, Science and History. The names of the twenty students are written on pieces of paper and then put into a bag.
(d)  One name is randomly selected from the bag. Calculate the probability that the student selected studies
   (i)  all three subjects;
   (ii)  History or Science.
(e)  A team of two students is to be randomly selected to compete in the quiz evening. The first student selected will be the captain of the team. Calculate the probability that
   (i)  the captain studies all three subjects and the other team member does not study any of the three subjects;
   (ii)  one student studies Science only and the other student studies History only;
(iii) the second student selected studies History, given that the captain studies History and Mathematics.

(a) (i) \(10 - 6 = 4\)
(ii) Students who study Mathematics only.
(iii) \(20 - (4 + 1 + 2 + 1 + 3 + 3 + 4) = 2\)

(b) \[
\begin{array}{ccc}
S & M & H \\
\hline
& & \\
\end{array}
\]

(c) \(n(S \cup H) = 8 + 9 - 3 = 14\)
OR
\(n(S \cup H) = 14\)

(d) (i) \(P(\text{studies all 3 subjects}) = \frac{2}{20} = \frac{1}{10} = 0.1\)
(ii) \(P(\text{History or Science}) = \frac{14}{20} = \frac{7}{10} = 0.7\)

(e) (i) \(P(\text{captain studies all, other student studies none}) = \frac{2}{20} \times \frac{2}{19} = \frac{4}{380} = \frac{1}{95} = 0.0105\)
(ii) \(P(\text{one studies Science only and the other studies History only}) = \frac{4}{20} \times \frac{3}{19} \times 2\)
(iii) \(P(\text{History} \mid \text{History and Maths}) = \frac{8}{19}\)

20. Heinrik rolls two 6-sided dice at the same time. One die has three red sides and three black sides. The other die has the sides numbered from 1 to 6. By means of a tree diagram, table of outcomes or otherwise, answer each of the following questions.
(a) How many different possible combinations can he roll?
(b) What is the probability that he will roll a red and an even number?
(c) What is the probability that he will roll a red or black and a 5?
(d) What is the probability that he will roll a number less than 3?

(a) 12
(b) \(\frac{3}{12} = \frac{1}{4}\) or 25%
(c) \(\frac{2}{12} = \frac{1}{6}\) or 16.7% (3 s.f.)
(d) \(\frac{4}{12} = \frac{1}{3}\) or 33.3% (3 s.f.)
21. Today Philip intends to go walking. The probability of good weather (G) is \( \frac{3}{4} \). If the weather is good, the probability he will go walking (W) is \( \frac{17}{20} \). If the weather forecast is not good (NG) the probability he will go walking is \( \frac{1}{5} \).

(a) Complete the probability tree diagram to illustrate this information.

(b) What is the probability that Philip will go walking? [8]

\[ P(G \cap W) = \frac{3}{4} \times \frac{17}{20} \]
\[ P(NG \cap W) = \frac{1}{4} \times \frac{1}{5} \]
\[ P(W) = \frac{3}{4} \times \frac{17}{20} + \frac{1}{4} \times \frac{1}{5} \]
\[ = \frac{11}{16} \] (0.6875, 68.75% or 0.688 to 3 s.f.)

22. A biased die with four faces is used in a game. A player pays 10 counters to roll the die. The table below shows the possible scores on the die, the probability of each score and the number of counters the player receives in return for each score.

<table>
<thead>
<tr>
<th>Score</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
<td>( \frac{1}{2} )</td>
<td>( \frac{1}{5} )</td>
<td>( \frac{1}{5} )</td>
<td>( \frac{1}{10} )</td>
</tr>
<tr>
<td>Number of counters player receives</td>
<td>4</td>
<td>5</td>
<td>15</td>
<td>( n )</td>
</tr>
</tbody>
</table>

Find the value of \( n \) in order for the player to get an expected return of 9 counters per roll. [4]

Let \( X \) be the number of counters the player receives in return.
\[ E(X) = \sum p(x) \times x = 9 \quad \Leftrightarrow \quad \left( \frac{1}{2} \times 4 \right) + \left( \frac{1}{5} \times 5 \right) + \left( \frac{1}{5} \times 15 \right) + \left( \frac{1}{10} \times n \right) = 9 \]

\[ \Leftrightarrow \frac{1}{10} n = 3 \quad \Rightarrow \quad n = 30 \]