1 Using a scale of 1 cm represents 10 units, sketch a vector to represent:
   a an aeroplane taking off at an angle of 8° to a runway with a speed of 60 m s⁻¹
   b a displacement of 45 m in a north-easterly direction.

2 Simplify:
   a \( \overrightarrow{AB} - \overrightarrow{CB} \)  
   b \( \overrightarrow{AB} + \overrightarrow{BC} - \overrightarrow{DC} \).

3 Construct vector equations for:

4 If \( \overrightarrow{PQ} = \begin{pmatrix} -4 \\ 1 \end{pmatrix} \), \( \overrightarrow{RQ} = \begin{pmatrix} -1 \\ 2 \end{pmatrix} \), and \( \overrightarrow{RS} = \begin{pmatrix} 2 \\ -3 \end{pmatrix} \), find \( \overrightarrow{SP} \).

5 [BC] is parallel to [OA] and is twice its length.
Find, in terms of \( p \) and \( q \), vector expressions for:
   a \( \overrightarrow{AC} \)  
   b \( \overrightarrow{OM} \).

6 Find \( m \) and \( n \) if \( \begin{pmatrix} 3 \\ m \end{pmatrix} \) and \( \begin{pmatrix} -12 \\ -20 \end{pmatrix} \) are parallel vectors.

7 If \( \overrightarrow{AB} = \begin{pmatrix} 2 \\ -7 \\ 4 \end{pmatrix} \) and \( \overrightarrow{AC} = \begin{pmatrix} -6 \\ 1 \\ -3 \end{pmatrix} \), find \( \overrightarrow{CB} \).

8 If \( \overrightarrow{p} = \begin{pmatrix} 3 \\ -2 \end{pmatrix} \), \( \overrightarrow{q} = \begin{pmatrix} -1 \\ 5 \end{pmatrix} \), and \( \overrightarrow{r} = \begin{pmatrix} -3 \\ 4 \end{pmatrix} \), find:
   a \( \overrightarrow{p} \cdot \overrightarrow{q} \)  
   b \( \overrightarrow{q} \cdot (\overrightarrow{p} - \overrightarrow{r}) \).

9 Consider points \( X(-2, 5) \), \( Y(3, 4) \), \( W(-3, -1) \), and \( Z(4, 10) \). Use vectors to show that WYZX is a parallelogram.

10 Consider points \( A(2, 3) \), \( B(-1, 4) \), and \( C(3, k) \). Find \( k \) if \( B\overrightarrow{A}C \) is a right angle.

11 Explain why:
   a \( \overrightarrow{a} \cdot \overrightarrow{b} \cdot \overrightarrow{c} \) is meaningless  
   b the expression \( \overrightarrow{a} \cdot \overrightarrow{b} \times \overrightarrow{c} \) does not need brackets.

12 Find all vectors which are perpendicular to the vector \( \begin{pmatrix} -4 \\ 5 \end{pmatrix} \).
13 In this question you may **not** assume any diagonal properties of parallelograms.

OABC is a parallelogram with \( \vec{OA} = p \) and \( \vec{OC} = q \). M is the midpoint of [AC].

**a** Find in terms of \( p \) and \( q \):

i. \( \vec{OB} \)

ii. \( \vec{OM} \)

**b** Hence show that O, M, and B are collinear, and that M is the midpoint of [OB].

14 Find the values of \( k \) such that the following are unit vectors:

a. \( \begin{pmatrix} 4 \\ 7 \\ 1 \end{pmatrix} \)

b. \( \begin{pmatrix} k \\ k \end{pmatrix} \)

15 Suppose \( |a| = 2 \), \( |b| = 4 \), and \( |c| = 5 \). Find:

a. \( a \cdot b \)

b. \( b \cdot c \)

c. \( a \cdot c \)

16 Find \( a \) and \( b \) if \( J(-4, 1, 3) \), \( K(2, -2, 0) \), and \( L(a, b, 2) \) are collinear.

17 Given \( |u| = 3 \), \( |v| = 5 \), and \( u \times v = \begin{pmatrix} 1 \\ -3 \\ -4 \end{pmatrix} \), find the possible values of \( u \cdot v \).

18 [AB] and [CD] are diameters of a circle with centre O.

**a** If \( \vec{OC} = q \) and \( \vec{OB} = r \), find:

i. \( \vec{DB} \) in terms of \( q \) and \( r \)

ii. \( \vec{AC} \) in terms of \( q \) and \( r \).

**b** What can be deduced about [DB] and [AC]?

19 **a** Find \( t \) given that \( \begin{pmatrix} 2 - t \\ 3 \\ t \end{pmatrix} \) and \( \begin{pmatrix} t \\ 4 - t \end{pmatrix} \) are perpendicular.

**b** Show that K(4, 3, -1), L(-3, 4, 2), and M(2, 1, -2) are vertices of a right angled triangle.
REVIEW SET 14B

1. Copy the given vectors and find geometrically:
   \[ \mathbf{a} \mathbf{x} + \mathbf{y} \quad \mathbf{b} \mathbf{y} - 2\mathbf{x} \]

2. Show that \( A(-2, -1, 3), \ B(4, 0, -1), \ \text{and} \ C(-2, 1, -4) \) are vertices of an isosceles triangle.

3. If \( \mathbf{r} = \begin{pmatrix} 4 \\ 1 \end{pmatrix} \) and \( \mathbf{s} = \begin{pmatrix} -3 \\ 2 \end{pmatrix} \) find: \( \mathbf{a} |\mathbf{s}| \quad \mathbf{b} |\mathbf{r} + \mathbf{s}| \quad \mathbf{c} |2\mathbf{s} - \mathbf{r}| \)

4. Find scalars \( r \) and \( s \) such that \( r \begin{pmatrix} -2 \\ 1 \end{pmatrix} + s \begin{pmatrix} 3 \\ -4 \end{pmatrix} = \begin{pmatrix} 13 \\ -24 \end{pmatrix} \).

5. Given \( \mathbf{P}(2, 3, -1) \) and \( \mathbf{Q}(-4, 4, 2) \), find:
   \( \mathbf{a} \) the distance between \( \mathbf{P} \) and \( \mathbf{Q} \)
   \( \mathbf{b} \) the midpoint of \( [\mathbf{PQ}] \)

6. If \( \mathbf{A}(4, 2, -1), \ \mathbf{B}(-1, 5, 2), \ \mathbf{C}(3, -3, c) \) are vertices of triangle \( \mathbf{ABC} \) which is right angled at \( \mathbf{B} \), find the value of \( c \).

7. Find the angle between the vectors \( \mathbf{a} = 3\mathbf{i} + \mathbf{j} - 2\mathbf{k} \) and \( \mathbf{b} = 2\mathbf{i} + 5\mathbf{j} + \mathbf{k} \).

8. Find two points on the \( Z \)-axis which are 6 units from \( \mathbf{P}(-4, 2, 5) \).

9. Determine all possible values of \( t \) if \( \begin{pmatrix} 3 \\ 3 - 2t \end{pmatrix} \) and \( \begin{pmatrix} t^2 + t \\ -2 \end{pmatrix} \) are perpendicular.

10. Prove that \( \mathbf{P}(-6, 8, 2), \ \mathbf{Q}(4, 6, 8), \ \text{and} \ \mathbf{R}(19, 3, 17) \) are collinear.

11. If \( \mathbf{u} = \begin{pmatrix} -4 \\ 2 \\ 1 \end{pmatrix} \) and \( \mathbf{v} = \begin{pmatrix} -1 \\ 3 \\ -2 \end{pmatrix} \), find: \( \mathbf{a} \) \( \mathbf{u} \cdot \mathbf{v} \) \quad \mathbf{b} \) the angle between \( \mathbf{u} \) and \( \mathbf{v} \).

12. If \( \mathbf{u} = \begin{pmatrix} 1 \end{pmatrix} \) and \( \mathbf{v} = \begin{pmatrix} 2 \end{pmatrix} \), find: \( \mathbf{a} \) \( \mathbf{u} + \mathbf{v} \) \quad \mathbf{b} \mathbf{u} \cdot \mathbf{v} \quad \mathbf{c} \mathbf{u} \times \mathbf{v} \)

13. \([\mathbf{AP}] \) and \([\mathbf{BQ}] \) are altitudes of triangle \( \mathbf{ABC} \).
   Let \( \overrightarrow{\mathbf{OA}} = \mathbf{p}, \ \overrightarrow{\mathbf{OB}} = \mathbf{q}, \ \text{and} \ \overrightarrow{\mathbf{OC}} = \mathbf{r}. \)
   \( \mathbf{a} \) Find vector expressions for \( \overrightarrow{\mathbf{AC}} \) and \( \overrightarrow{\mathbf{BC}} \) in terms of \( \mathbf{p}, \ \mathbf{q}, \ \text{and} \ \mathbf{r}. \)
   \( \mathbf{b} \) Using the property \( \mathbf{a} \cdot (\mathbf{b} - \mathbf{c}) = \mathbf{a} \cdot \mathbf{b} - \mathbf{a} \cdot \mathbf{c}, \)
   deduce that \( \mathbf{q} \cdot \mathbf{r} = \mathbf{p} \cdot \mathbf{q} = \mathbf{p} \cdot \mathbf{r}. \)
   \( \mathbf{c} \) Hence prove that \([\mathbf{OC}] \) is perpendicular to \([\mathbf{AB}] \).
14  Find two vectors of length 4 units which are parallel to \(3\mathbf{i} - 2\mathbf{j} + \mathbf{k}\).

15  Find the measure of \(\overline{DMC}\).

16  Find all vectors perpendicular to both \(\begin{pmatrix} 2 \\ -1 \\ -2 \end{pmatrix}\) and \(\begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}\).

17  a  Find \(k\) given that \(\begin{pmatrix} k \\ \frac{1}{\sqrt{2}} \\ -k \end{pmatrix}\) is a unit vector.

   b  Find the vector which is 5 units long and has the opposite direction to \(\begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix}\).

18  Find the angle between \(\begin{pmatrix} 2 \\ -4 \\ 3 \end{pmatrix}\) and \(\begin{pmatrix} -1 \\ 1 \\ 3 \end{pmatrix}\).

19  Determine the measure of \(\overline{QDM}\) given that M is the midpoint of \([PS]\).
5 In the figure alongside, \[ \overrightarrow{OP} = p, \quad \overrightarrow{OR} = r, \quad \text{and} \quad \overrightarrow{RQ} = q. \] M and N are the midpoints of [PQ] and [QR] respectively.

Find, in terms of \( p, q, \) and \( r \):

\[ \begin{align*}
\mathbf{a} & \quad \overrightarrow{OQ} \\
\mathbf{b} & \quad \overrightarrow{PQ} \\
\mathbf{c} & \quad \overrightarrow{ON} \\
\mathbf{d} & \quad \overrightarrow{MN}
\end{align*} \]

6 Suppose \( p = \begin{pmatrix} -3 \\ 2 \end{pmatrix}, \ q = \begin{pmatrix} 2 \\ -4 \end{pmatrix}, \) and \( r = \begin{pmatrix} 3 \\ 0 \end{pmatrix}. \) Find \( x \) if:

\[ \begin{align*}
\mathbf{a} & \quad p - 3x = 0 \\
\mathbf{b} & \quad 2q - x = r
\end{align*} \]

7 Suppose \( |v| = 3 \) and \( |w| = 2. \) If \( v \) is parallel to \( w, \) what values might \( v \cdot w \) take?

8 Find a unit vector which is parallel to \( \mathbf{i} + \mathbf{j} - 2\mathbf{k} \) and perpendicular to \( 2\mathbf{i} + 2\mathbf{j} - \mathbf{k}. \)

9 Find \( t \) if \( \begin{pmatrix} t \\ -4 \\ t + 2 \end{pmatrix} \) and \( \begin{pmatrix} t \\ 1 + t \\ -3 \end{pmatrix} \) are perpendicular vectors.

10 Find all angles of the triangle with vertices \( K(3, 1, 4), \ L(-2, 1, 3), \) and \( M(4, 1, 3). \)

11 Find \( k \) if the following are unit vectors:

\[ \begin{align*}
\mathbf{a} & \quad \begin{pmatrix} \frac{5}{3} \\ k \end{pmatrix} \\
\mathbf{b} & \quad \begin{pmatrix} k \\ k \end{pmatrix}
\end{align*} \]

12 Use vector methods to find the measure of \( \angle GAC \) in the rectangular box alongside.

13 Using \( p = \begin{pmatrix} 3 \\ -2 \end{pmatrix}, \ q = \begin{pmatrix} -2 \\ 5 \end{pmatrix}, \) and \( r = \begin{pmatrix} 1 \\ -3 \end{pmatrix}, \) verify that:

\[ p \cdot (q - r) = p \cdot q - p \cdot r. \]

14 P\((-1, 2, 3)\) and Q\((4, 0, -1)\) are two points in space. Find:

\[ \begin{align*}
\mathbf{a} & \quad \overrightarrow{PQ} \\
\mathbf{b} & \quad \text{the angle that} \ \overrightarrow{PQ} \ \text{makes with the} \ X\text{-axis.}
\end{align*} \]

15 Suppose \( \overrightarrow{OM} = \begin{pmatrix} -2 \\ 4 \end{pmatrix}, \ \overrightarrow{MP} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}, \ \overrightarrow{MP} \cdot \overrightarrow{PT} = 0, \) and \( |\overrightarrow{MP}| = |\overrightarrow{PT}|. \)

Write down the two possible position vectors \( \overrightarrow{OT}. \)

16 Given \( p = 2\mathbf{i} - \mathbf{j} + 4\mathbf{k} \) and \( q = -\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}, \) find the angle between \( p \) and \( q. \)

17 Suppose \( u = 2\mathbf{i} + \mathbf{j}, \ v = 3\mathbf{j}, \) and \( \theta \) is the acute angle between \( u \) and \( v. \)

Find the exact value of \( \sin \theta. \)

18 Find two vectors of length 3 units which are perpendicular to both \(-\mathbf{i} + 3\mathbf{k}\) and \(2\mathbf{i} - \mathbf{j} + \mathbf{k}. \)
1. For the line that passes through \((-6, 3)\) with direction \(\begin{pmatrix} 4 \\ -3 \end{pmatrix}\), write down the corresponding:
   a. vector equation
   b. parametric equations
   c. Cartesian equation.

2. \((-3, m)\) lies on the line with vector equation \(\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 18 \\ -2 \end{pmatrix} + t \begin{pmatrix} -7 \\ 4 \end{pmatrix}\). Find \(m\).

3. Line \(L\) has equation \(r = \begin{pmatrix} 3 \\ -3 \end{pmatrix} + t \begin{pmatrix} 2 \\ 5 \end{pmatrix}\).
   a. Locate the point on the line corresponding to \(t = 1\).
   b. Explain why the direction of the line could also be described by \(\begin{pmatrix} 4 \\ 10 \end{pmatrix}\).
   c. Use your answers to a and b to write an alternative vector equation for line \(L\).

4. \(P(2, 0, 1)\), \(Q(3, 4, -2)\), and \(R(-1, 3, 2)\) are three points in space.
   a. Find parametric equations of line \((PQ)\).
   b. Show that if \(\theta = \overrightarrow{PQ}, \) then \(\cos \theta = -\frac{20}{\sqrt{260}\sqrt{33}}\).

5. Triangle \(ABC\) is formed by three lines:
   Line \((AB)\) is \(\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ -1 \end{pmatrix} + t \begin{pmatrix} 1 \\ 3 \end{pmatrix}\). Line \((BC)\) is \(\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 7 \\ 4 \end{pmatrix} + s \begin{pmatrix} 1 \\ -1 \end{pmatrix}\).
   Line \((AC)\) is \(\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \end{pmatrix} + u \begin{pmatrix} 3 \\ 1 \end{pmatrix}\). \(s, t,\) and \(u\) are scalars.
   a. Use vector methods to find the coordinates of \(A, B,\) and \(C\).
   b. Find \(|\overrightarrow{AB}|, |\overrightarrow{BC}|,\) and \(|\overrightarrow{AC}|\).
   c. Classify triangle \(ABC\).

6. a. Consider two unit vectors \(\mathbf{a}\) and \(\mathbf{b}\). Prove that the vector \(\mathbf{a} + \mathbf{b}\) bisects the angle between vector \(\mathbf{a}\) and vector \(\mathbf{b}\).
   b. Consider the points \(H(9, 5, -5), J(7, 3, -4),\) and \(K(1, 0, 2)\).
      Find the equation of the line \(L\) that passes through \(J\) and bisects \(HK\).
   c. Find the coordinates of the point where \(L\) meets \((HK)\).

7. Suppose \(A\) is \((3, 2, -1)\) and \(B\) is \((-1, 2, 4)\).
   a. Write down a vector equation of the line through \(A\) and \(B\).
   b. Find the equation of the plane through \(B\) with normal \(\overrightarrow{AB}\).
   c. Find two points on \((AB)\) which are \(2\sqrt{11}\) units from \(A\).

8. For \(C(-3, 2, -1)\) and \(D(0, 1, -4)\), find the coordinates of the point where the line passing through \(C\) and \(D\) meets the plane with equation \(2x - y + z = 3\).
9 Suppose \( \overrightarrow{OA} = \mathbf{a}, \overrightarrow{OB} = \mathbf{b}, \ |\mathbf{a}| = 3, \ |\mathbf{b}| = \sqrt{7}, \) and \( \mathbf{a} \times \mathbf{b} = \mathbf{i} + 2\mathbf{j} - 3\mathbf{k} \). Find:
   
   a  \( \mathbf{a} \cdot \mathbf{b} \)  
   b  the area of triangle OAB.

10 a  How far is \( X(-1, 1, 3) \) from the plane \( x - 2y - 2z = 8 \)?
   b  Find the coordinates of the foot of the perpendicular from \( Q(-1, 2, 3) \) to the line \( 2 - x = y - 3 = z \).

11 a  Find if possible the point where the line through \( L(1, 0, 1) \) and \( M(-1, 2, -1) \) meets the plane with equation \( x - 2y - 3z = 14 \).
   b  Find the shortest distance from \( L \) to the plane.

12 The equations of two lines are:
   \[ L_1: \ x = 3t - 4, \ y = t + 2, \ z = 2t + 1 \]
   \[ L_2: \ x = \frac{y - 5}{2} = \frac{-z - 1}{2} \]
   a  Find the point of intersection of \( L_1 \) and the plane \( 2x + y - z = 2 \).
   b  Find the point of intersection of \( L_1 \) and \( L_2 \).
   c  Find the equation of the plane that contains \( L_1 \) and \( L_2 \).

13 Show that the line \( x - 1 = \frac{y + 2}{2} = \frac{z - 3}{4} \) is parallel to the plane \( 6x + 7y - 5z = 8 \) and find the distance between them.

14 \( x^2 + y^2 + z^2 = 26 \) is the equation of a sphere with centre \((0, 0, 0)\) and radius \( \sqrt{26} \) units.
   Find the point(s) where the line through \((3, -1, -2)\) and \((5, 3, -4)\) meets the sphere.

15 When an archer fires an arrow, he is suddenly aware of a breeze which pushes his shot off-target. The speed of the shot \( |\mathbf{v}| \) is not affected by the wind, but the arrow’s flight is \( 2^\circ \) off-line.
   a  Draw a vector diagram to represent the situation.
   b  Hence explain why:
      i  the breeze must be \( 91^\circ \) to the intended direction of the arrow
      ii  the speed of the breeze must be \( 2 |\mathbf{v}| \sin 1^\circ \).

16 In the figure \( ABCD \) is a parallelogram. \( X \) is the midpoint of \( BC \), and \( Y \) is on \([AX]\) such that \( \overrightarrow{AY} = 2\overrightarrow{YX} \).
   a  Find the coordinates of \( X \) and \( D \).
   b  Find the coordinates of \( Y \).
   c  Show that \( B, Y, \) and \( D \) are collinear.

17 Solve the system \[
\begin{align*}
   x - y + z &= 5 \\
   2x + y - z &= -1 \\
   7x + 2y + kz &= -k
\end{align*}
\]
   for any real number \( k \)
   using row operations.
   Give geometric interpretations of your results.
1 Find the vector equation of the line which cuts the y-axis at (0, 8) and has direction \(5\mathbf{i} + 4\mathbf{j}\).

2 A yacht is sailing with constant speed \(5\sqrt{10}\) km h\(^{-1}\) in the direction \(-\mathbf{i} - 3\mathbf{j}\). Initially it is at point \((-6, 10)\). A beacon is at \((0, 0)\) at the centre of a tiny atoll. Distances are in kilometres.
   a Find in terms of \(\mathbf{i}\) and \(\mathbf{j}\):
      i the initial position vector of the yacht
      ii the direction vector of the yacht
      iii the position vector of the yacht at any time \(t\) hours, \(t \geq 0\).
   b Find the time when the yacht is closest to the beacon.
   c If there is a reef of radius 8 km around the atoll, will the yacht hit the reef?

3 Write down
   i a vector equation
   ii parametric equations for the line passing through:
   a \((2, -3)\) with direction \(\begin{pmatrix} 4 \\ -1 \end{pmatrix}\)
   b \((-1, 6, 3)\) and \((5, -2, 0)\).

4 A small plane can fly at 350 km h\(^{-1}\) in still conditions. Its pilot needs to fly due north, but needs to deal with a 70 km h\(^{-1}\) wind from the east.
   a In what direction should the pilot face the plane in order that his resultant velocity is due north?
   b What will the speed of the plane be?

5 Find the angle between line \(L_1\) passing through \((0, 3)\) and \((5, -2)\), and line \(L_2\) passing through \((-2, 4)\) and \((-6, 7)\).

6 Submarine X23 is at \((2, 4)\). It fires a torpedo with velocity vector \(\begin{pmatrix} 1 \\ -3 \end{pmatrix}\) at exactly 2:17 pm.
   Submarine Y18 is at \((11, 3)\). It fires a torpedo with velocity vector \(\begin{pmatrix} -1 \\ a \end{pmatrix}\) at 2:19 pm to intercept the torpedo from X23. Distance units are kilometres. \(t\) is in minutes.
   a Find \(x_1(t)\) and \(y_1(t)\) for the torpedo fired from submarine X23.
   b Find \(x_2(t)\) and \(y_2(t)\) for the torpedo fired from submarine Y18.
   c At what time does the interception occur?
   d What was the direction and speed of the interception torpedo?

7 Suppose \(P_1\) is the plane \(2x - y - 2z = 9\) and \(P_2\) is the plane \(x + y + 2z = 1\).
   \(L\) is the line with parametric equations \(x = t,\ y = 2t - 1,\ z = 3 - t\).
   Find the acute angle between:
   a \(L\) and \(P_1\)
   b \(P_1\) and \(P_2\).
8 Consider the lines \( L_1: \frac{x-8}{3} = \frac{y+9}{-16} = \frac{z-10}{7} \) and \( L_2: x = 15 + 3t, \ y = 29 + 8t, \ z = 5 - 5t \).
   a) Show that the lines are skew.
   b) Find the acute angle between them.
   c) Line \( L_3 \) is a translation of \( L_1 \) which intersects \( L_2 \). Find the equation of the plane containing \( L_2 \) and \( L_3 \).
   d) Find the shortest distance between them.

9 a) Find the equation of the plane through \( A(-1, 2, 3), \ B(1, 0, -1) \), and \( C(0, -1, 5) \).
   b) If \( X \) is \( (3, 2, 4) \), find the angle that \( AX \) makes with this plane.

10 a) Find all vectors of length 3 units which are normal to the plane \( x - y + z = 6 \).
   b) Find a unit vector parallel to \( i + rj + 3k \) and perpendicular to \( 2i - j + 2k \).
   c) The distance from \( A(-1, 2, 3) \) to the plane with equation \( 2x - y + 2z = k \) is 3 units. Find \( k \).

11 Find the angle between the lines with equations \( 4x - 5y = 11 \) and \( 2x + 3y = 7 \).

12 Consider \( A(2, -1, 3) \) and \( B(0, 1, -1) \).
   a) Find the vector equation of the line through \( A \) and \( B \).
   b) Hence find the coordinates of \( C \) on \( AB \) which is 2 units from \( A \).

13 Find the angle between the plane \( 2x + 2y - z = 3 \) and the line \( x = t - 1, \ y = -2t + 4, \ z = -t + 3 \).

14 Let \( r = 2i - 2j - k, \ s = 2i + j + 2k, \ t = i + 2j - k \), be the position vectors of the points \( R, S \), and \( T \), respectively. Find the area of the triangle \( RST \).

15 Classify the following line pairs as either parallel, intersecting, or skew. In each case find the measure of the acute angle between them.
   a) \( x = 2 + t, \ y = -1 + 2t, \ z = 3 - t \) and \( x = -8 + 4s, \ y = s, \ z = 7 - 2s \)
   b) \( x = 3 + t, \ y = 5 - 2t, \ z = -1 + 3t \) and \( x = 2 - s, \ y = 1 + 3s, \ z = 4 + s \)

16 \( \mathbf{p} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} \) and \( \mathbf{q} = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} \)
   a) Find \( \mathbf{p} \times \mathbf{q} \).
   b) Find \( m \) if \( \mathbf{p} \times \mathbf{q} \) is perpendicular to the line \( \mathbf{L} \) with equation \( \mathbf{r} = \begin{pmatrix} -2 \\ 3 \\ \lambda \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ m \end{pmatrix} \).
   c) Hence find the equation of the plane \( P \) containing \( \mathbf{L} \) which is perpendicular to \( \mathbf{p} \times \mathbf{q} \).
   d) Find \( t \) if the point \( A(4, t, 2) \) lies on the plane \( P \).
   e) For the value of \( t \) found in d, if \( B \) is the point \( (6, -3, 5) \), find the exact value of the sine of the angle between \( \mathbf{AB} \) and the plane \( P \).

17 a) Show that the plane \( 2x + y + z = 5 \) contains the line \( L_1: x = -2t + 2, \ y = t, \ z = 3t + 1, \ t \in \mathbb{R} \).
   b) For what values of \( k \) does the plane \( x + ky + z = 3 \) contain \( L_1 \)?
   c) Hence find the values of \( p \) and \( q \) for which the following system of equations has an infinite number of solutions. Clearly explain your reasoning.

\[
\begin{align*}
2x + y + z &= 5 \\
x - y + z &= 3 \\
-2x + py + 2z &= q
\end{align*}
\]
Consider the system\[\begin{align*}
x - 3y + 2z &= -5 \\
3x + y + (2 - k)z &= 10 \\
-2x + 6y + kz &= 5
\end{align*}\]
where \(k\) can take any real value.

a Reduce the system to echelon form.

b For what value of \(k\) does the system have no solutions? Interpret this result geometrically.

c i For what value(s) of \(k\) does the system have a unique solution?

ii Find the unique solution in terms of \(k\), and interpret the result geometrically.

iii Find the unique solution when \(k = 1\).

**REVIEW SET 15C**

1 Find the velocity vector of an object moving in the direction \(3i - j\) with speed 20 km h\(^{-1}\).

2 A moving particle has coordinates \(P(x(t), y(t))\) where \(x(t) = -4 + 8t\) and \(y(t) = 3 + 6t\). The distance units are metres, and \(t \geq 0\) is the time in seconds. Find the:

a initial position of the particle

b position of the particle after 4 seconds

c particle’s velocity vector

d speed of the particle.

3 Trapezium KLMN is formed by the following lines:

\begin{align*}
\text{(KL)}\quad &\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 19 \end{pmatrix} + p \begin{pmatrix} 5 \\ -2 \end{pmatrix} \\
\text{(ML)}\quad &\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 33 \\ -5 \end{pmatrix} + q \begin{pmatrix} -11 \\ 16 \end{pmatrix} \\
\text{(NK)}\quad &\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \\ 7 \end{pmatrix} + r \begin{pmatrix} 4 \\ 10 \end{pmatrix} \\
\text{(MN)}\quad &\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 43 \\ -9 \end{pmatrix} + s \begin{pmatrix} -5 \\ 2 \end{pmatrix}
\end{align*}

\(p, q, r,\) and \(s\) are scalars.

a Which two lines are parallel? Explain your answer.

b Which lines are perpendicular? Explain your answer.

c Use vector methods to find the coordinates of K, L, M, and N.

d Calculate the area of trapezium KLMN.

4 Find the angle between the lines:

\(L_1:\ x = 1 - 4t, \quad y = 3t\) and \(L_2:\ x = 2 + 5s, \quad y = 5 - 12s\).

5 Consider \(A(3, -1, 1)\) and \(B(0, 2, -2)\).

a Find \(|\overrightarrow{AB}|\).

b Show that the line passing through A and B can be described by \(\mathbf{r} = 2\mathbf{j} - 2\mathbf{k} + \lambda(-1 + \mathbf{j} - \mathbf{k})\) where \(\lambda\) is a scalar.

c Find the angle between \((\overrightarrow{AB})\) and the line with vector equation \(t(\mathbf{i} + \mathbf{j} + \mathbf{k})\).
Let \( \mathbf{i} \) represent a displacement 1 km due east and \( \mathbf{j} \) represent a displacement 1 km due north.

Road A passes through \((-9, 2)\) and \((15, -16)\).
Road B passes through \((6, -18)\) and \((21, 18)\).

a Find a vector equation for each of the roads.
b An injured hiker is at \((4, 11)\), and needs to travel the shortest possible distance to a road. Towards which road should he head, and how far will he need to walk to reach this road?

Given the points \( A(4, 2, -1), B(2, 1, 5), \) and \( C(9, 4, 1) \):

a Show that \( \overrightarrow{AB} \) is perpendicular to \( \overrightarrow{AC} \).
b Find the equation of the line through: i A and B ii A and C.

The triangle with vertices \( P(-1, 2, 1), Q(0, 1, 4), \) and \( R(a, -1, -2) \) has area \( \sqrt{118} \) units\(^2\). Find \( a \).

Consider \( A(-1, 2, 3), B(2, 0, -1), \) and \( C(-3, 2, -4) \).

a Find the equation of the plane defined by \( A, B, \) and \( C \).
b Find the measure of \( \angle CAB \).
c \( D(r, 1, -r) \) is a point such that \( \overrightarrow{BD} \) is a right angle. Find \( r \).

Given \( A(-1, 2, 3), B(1, 0, -1), \) and \( C(1, 3, 0) \), find:
a the normal vector to the plane containing \( A, B, \) and \( C \)
b \( D \), the fourth vertex of parallelogram \( ACBD \)
c the area of parallelogram \( ACBD \)
d the coordinates of the foot of the perpendicular from \( C \) to the line \( AB \).

P(2, 0, 1), Q(3, 4, -2), and R(-1, 3, 2) are three points in space. Find:
a \( \overrightarrow{PQ}, \overrightarrow{QR} \), and \( \overrightarrow{QR} \) the parametric equations of \( \overrightarrow{PQ} \)
b a vector equation of the plane \( PQR \).

Given the point \( A(-1, 3, 2) \), the plane \( 2x - y + 2z = 8 \), and the line defined by \( x = 7 - 2t, y = -6 + t, z = 1 + 5t \), find:
a the distance from \( A \) to the plane
b the coordinates of the point on the plane nearest to \( A \)
c the shortest distance from \( A \) to the line.

a Find the equation of the plane through \( A(-1, 0, 2), B(0, -1, 1), \) and \( C(1, 2, -1) \).
b Find the equation of the line, in parametric form, which passes through the origin and is normal to the plane in a.
c Find the point where the line in b intersects the plane in a.
14 Consider the lines with equations \( \frac{x-3}{2} = y - 4 = \frac{z+1}{-2} \) and 
\( x = -1 + 3t, \ y = 2 + 2t, \ z = 3 - t. \)
   
   a Are the lines parallel, intersecting, or skew? Justify your answer.
   b Determine the acute angle between the lines.

15 Line 1 has equation \( \frac{x-8}{3} = \frac{y+9}{-16} = \frac{z-10}{7}. \)
Line 2 has vector equation \( \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 15 \\ 29 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 8 \\ -5 \end{pmatrix}. \)
   
   a Show that lines 1 and 2 are skew.
   b Line 3 is a translation of line 2. Find the equation of the plane containing lines 2 and 3.
   c Hence find the shortest distance between lines 1 and 2.
   d Find the coordinates of the points where the common perpendicular meets the lines 1 and 2.

16 Lines \( L_1 \) and \( L_2 \) are defined by
\( L_1: \ \mathbf{r} = \begin{pmatrix} 3 \\ -2 \\ -2 \end{pmatrix} + s \begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix} \) and \( L_2: \ \mathbf{r} = \begin{pmatrix} 3 \\ 0 \\ -1 \end{pmatrix} + t \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix}. \)
   
   a Find the coordinates of A, the point of intersection of the lines.
   b Show that the point \( B(0, -3, 2) \) lies on the line \( L_2 \).
   c Find the equation of the line BC given that \( C(3, -2, -2) \) lies on \( L_1 \).
   d Find the equation of the plane containing A, B, and C.
   e Find the area of triangle ABC.
   f Show that the point \( D(9, -4, 2) \) lies on the normal to the plane passing through C.

17 Three planes have the equations given below:
   \( \text{Plane } A: \ x + 3y + 2z = 5 \)
   \( \text{Plane } B: \ 2x + y + 9z = 20 \)
   \( \text{Plane } C: \ x - y + 6z = 8 \)
   
   a Show that plane \( A \) and plane \( B \) intersect in a line \( L_1 \).
   b Show that plane \( B \) and plane \( C \) intersect in a line \( L_2 \).
   c Show that plane \( A \) and plane \( C \) intersect in a line \( L_3 \).
   d Show that \( L_1, L_2, \) and \( L_3 \) are parallel but not coincident.
   e What does this mean geometrically?
REVIEW SET 14A

1 a

If
\[ t = \frac{5}{4} \]
then \( t \neq 0 \).

12 a \( \overrightarrow{AC} = \frac{p + q}{2} \)

13 a \( \overrightarrow{LQ} = \frac{t}{2} \)

14 a \( k = \pm \frac{7}{\sqrt{50}} \)

15 a a \cdot b = -4

16 a = -2, b = 0

17 If \( \theta \) is acute, \( u \cdot v = \sqrt{199} \); if \( \theta \) is obtuse, \( u \cdot v = -\sqrt{199} \).

18 a \( \frac{5}{3} \)

b \( \frac{3}{2} \)

19 a \( t = -4 \)

b \( \frac{5}{3} \)

\( \overrightarrow{LM} = \frac{-3}{-4} \).

\( \overrightarrow{KM} = \frac{-2}{-1} \).

\( \theta = 90^\circ \).

\( \overrightarrow{LM} = \frac{5}{3} \).

\( \overrightarrow{KM} = \frac{-2}{-1} \).

\( \theta = 90^\circ \).

REVIEW SET 14B

1 a

2 a \( \overrightarrow{AC} = \frac{\overrightarrow{AD}}{5} \)

3 a \( q = p + r \)

b \( l = k - j + n - m \)

4 \( \overrightarrow{AB} = \overrightarrow{AC} \) and \( \overrightarrow{BC} = \overrightarrow{CD} \)

\( \therefore \) \( \triangle \) is isosceles.

5 a \( \sqrt{13} \) units

b \( \sqrt{16} \) units

\( c \) \( \sqrt{109} \) units

6 \( r = 4, \quad s = 7 \)

7 \( \overrightarrow{AC} = -\frac{6}{1}, \quad \overrightarrow{BC} = -\frac{3}{2} \)

\( \overrightarrow{CD} = 64.0^\circ \)

8 \( (0, 0, 1) \) and \( (0, 0, 9) \)

9 \( k = 6 \)

10 \( t = \frac{9}{4} \) or \(-3 \)

12 a \( 8 \) b \( \approx 62.2^\circ \)

13 a \( \overrightarrow{AC} = -p + r, \quad \overrightarrow{BC} = -q + r \)

14 \( \pm \frac{1}{\sqrt{26}} (3i - 2j + k) \)

15 \( \approx 16.1^\circ \)

16 \( \left( \frac{0}{-2}, 1 \right) \)

17 \( k = \pm \frac{1}{2} \)

b \( \pm \frac{3}{-1} \)

18 \( \approx 80.3^\circ \)

19 \( \approx 26.4^\circ \)

REVIEW SET 14C

1 a \( \overrightarrow{PQ} \)

b \( \overrightarrow{PR} \)

2 a \( \left( \frac{3}{11} \right) \)

b \( \left( \frac{7}{-3} \right) \)

\( \overrightarrow{CD} = \frac{1}{2} [\overrightarrow{AB}] \parallel [\overrightarrow{CD}] \)

b \( C \) is the midpoint of \( [AB] \).

3 a \( AB = \frac{1}{2} \overrightarrow{CD} \)

\( [AB] \parallel [CD] \)

b \( \overrightarrow{PQ} = \frac{3}{-12} \)

\( \overrightarrow{QD} = \frac{\sqrt{162}}{\overrightarrow{CD}} \)

\( \overrightarrow{QD} = \frac{\sqrt{61}}{\overrightarrow{CD}} \)

5 a \( r + q \)

b \( -p + r + q \)

c \( r + \frac{3}{2} q \)

d \( -\frac{1}{2} p + \frac{1}{2} r \)
They do not meet, the line is parallel to the plane.

12 a \( \left( \frac{1}{3}, \frac{1}{2}, \frac{2}{3} \right) \)  
\( b \) \((-1, 3, 1)\)  
\( c \) \(6x - 8y - 5z = -35\)
b \( \left( -2 - \frac{1}{\sqrt{3}}, -1 + \frac{2}{\sqrt{3}}, 3 - \frac{3}{\sqrt{3}} \right) \) and \\
\( \left( 2 + \frac{2}{\sqrt{3}}, -1 - \frac{2}{\sqrt{3}}, 3 + \frac{4}{\sqrt{3}} \right) \)

13. \( 7.82^2 = 14 \) units

14. \( 2\sqrt{2} \) units

15. a intersecting at (4, 3, 1), angle \( \approx 44.5^\circ \)

16. a) \( t = 2 \) 

b) \( k = 1 \) 

c) \( t = 11 \) units, \( \angle \approx 71.2^\circ \)

17. a) \( \bar{PQ} = \left( \frac{1}{4}, \frac{1}{4} \right) \), \( |\bar{PQ}| = \sqrt{25} \) units, \( \bar{QR} = \left( \frac{3}{3}, 2, \frac{3}{3} \right) \)

b) \( x = 2 + \lambda, \ y = 4\lambda, \ z = 1 - 3\lambda, \ \lambda \in \mathbb{R} \)

c) \( \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 4 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} -1 \\ 4 \end{pmatrix}, \ \lambda, \mu \in \mathbb{R} \)

12. a) \( (1, 2, 4) \) 

b) \( \theta \approx 27.0^\circ \)

15. a) \( t = 10, \ y = 6, \ z = 2 \) 

b) \( x = 5t, \ y = t, \ z = 4t, \ t \in \mathbb{R} \)

c) \( \left( \frac{6}{3}, \frac{3}{3}, \frac{3}{3} \right) \)

14. a) intersecting at (1, 2, 3) 

b) \( \theta \approx 27.0^\circ \)

16. a) \( A(2, -1, 0) \) 

b) A normal is \( \begin{pmatrix} 3 \\ 2 \\ -2 \end{pmatrix} \)

c) \( \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} \)

17. e The three planes have no common point of intersection. The line of intersection of any two planes is parallel to the third plane.