Topic 2: Mechanics
**Classical Mechanics:** - Study of the motion of macroscopic objects and causes of that motion and related concepts of force and energy

*Kinematics* – is concerned with the description of how objects move; their motion is described in terms of displacement, velocity, and acceleration.

*Dynamics* – explains why objects change the motion; explains changes using concepts of force and energy.
Displacement – vector

Distance = length of the curved line
Displacement is the distance moved in a particular direction.
It is an object’s change in position.
Displacement: \( \Delta x = x_2 - x_1 \)

Distance = length of the curved line

Velocity is the rate of change of displacement. vector \( (v) = m \text{s}^{-1} \)

\[
\text{velocity} = \frac{\text{change of displacement}}{\text{time taken}}
\]

Speed is the rate of change of distance. Scalar \( (v) = m \text{s}^{-1} \)

\[
\text{speed} = \frac{\text{distance covered}}{\text{time taken}}
\]

Acceleration is the rate of change of velocity.

\[
\text{acceleration} = \frac{\text{change of velocity}}{\text{time taken}}
\]
a = 3 m/s\(^2\) means that velocity changes 3 m/s every second!!!!!!

If an object’s initial velocity is 4 m/s then after one second it will be 7 m/s, after two seconds 10 m/s, ....

Acceleration can cause: 1. speeding up 2. slowing down 3. and/or changing direction

So beware: both velocity and acceleration are vectors. Therefore
1. if velocity and acceleration are in the same direction, speed of the body is increasing.
2. if velocity and acceleration are in the opposite directions, speed of the body is decreasing.
3. If a car changes direction even at constant speed it is accelerating. Why? Because the direction of the car is changing and therefore its velocity is changing. If its velocity is changing then it must have acceleration. There must be a force acting on the car

Velocity vector of a particle moving in a circle with speed 10 m/s at two separate points. The velocity vector is tangential to the circle.

Average acceleration is:

\[
\tilde{a} = \frac{\Delta \dot{v}}{t}
\]

\[
a = \frac{14.1 \text{ m/s}}{2s} = 7 \text{ m/s}^2
\]

\[
\tilde{a} = 7 \text{ m/s}^2, 225^0 \text{ (or} 45^0 \text{ SW)}
\]
Average and Instantaneous velocity and speed

**Instantaneous velocity** is the velocity of something at a specific time. There is no formula for it using algebra. Only if one knows $x(t)$, then derivative/slope of it would give $v(t)$.

**Instantaneous speed** is the speed at a specific time.

The speedometer of a car reveals information about the instantaneous speed of your car. It shows your speed at a particular instant in time. If direction is included you have instantaneous velocity.

**Average velocity** and **average speed** is calculated over a period of time.

Instantaneous speed is the magnitude of the instantaneous velocity.

BUT

Only if distance traveled is equal to magnitude of displacement, average speed is equal to magnitude of average velocity.
Frames of reference

A frame of reference is needed to determine either position or velocity of an object.

Both vehicle move forward relative to the stationary tree (when ground is frame of reference). Red car travelling at 30 m/s relative to the tree, travels at +5 m/s relative to silver car, so the gap between cars increases by 5 m/s. On the other hand red car travels at −45 m/s relative to blue car.
What can we find from graphs of motion?
What is:

1. the total distance travelled by the object during the 10.0 second time interval?
2. the displacement covered by the object during the 10.0 second time interval?

Total distance = \(8\text{m} + 8\text{m} + 8\text{m} = 24\text{ m}\)
Displacement = \(8\text{m}\), away from initial position
Interpreting motion graphs

- The **area** under a velocity-time graph is the displacement.
- The **area** under an acceleration-time graph is the change in velocity.

- Velocity is a **slope** of displacement – time graph.
- Acceleration is a **slope** of velocity – time graph.

![Graphs showing displacement, velocity, and acceleration](image-url)
• Velocity is gradient of displacement–time graph

Instantaneous velocities at P and T

\[ v_P = 2 \text{ m/s} \]
\[ v_T = -0.52 \text{ m/s} \]

Average velocity from P to T

\[ v_{\text{avg}} = \frac{24 - 18}{29 - 8} \approx 0.3 \text{ m/s} \]
- The gradient of a velocity–time graph is the acceleration.

\[ a = \frac{\Delta v}{\Delta t} \]

1s – 5s: \( a = \frac{50}{5} = 10 \text{ m/s}^2 \)  
velocity increasing

5s – 7s: \( a = \frac{0}{2} = 125 \text{ m} \)  
velocity is constant

7s – 8s: \( a = \frac{-50}{1} = -50 \text{ m/s}^2 \)  
velocity decreasing

- The area under a velocity-time graph is the displacement.

\[ s = \frac{1}{2} \times v \times t \]

1s – 5s: \( s = \frac{1}{2} \times 50 \times 5 = 125 \text{ m} \)

5s – 7s: \( s = 50 \times 2 = 100 \text{ m} \)

7s – 8s: \( s = \frac{1}{2} \times 50 \times 1 = 25 \text{ m} \)
The area under a acceleration-time graph is change in velocity.

The acceleration vs. time graph for a car starting from rest. Calculate the velocity of the car and hence draw the velocity vs. time graph.

0 s – 2 s:
\[ \Delta v = \left(2 \frac{m}{s^2}\right) (2s) = 4 \, m/s \]

2 s – 4 s:
\[ \Delta v = 0 \, m/s \]

4 s – 6 s:
\[ \Delta v = \left(-2 \frac{m}{s^2}\right) (2s) = -4 \, m/s \]

The acceleration had a negative value, which means that the velocity is decreasing. It starts at a velocity of 4 m/s and decreases to 0 m/s.
Kinematic equations of motion for uniform acceleration

$a$ is constant

\[ x = ut + \frac{1}{2}at^2 \]  
\textit{displacement covered in time } t \\
\[ v = u + at \]  
\textit{velocity after time } t \\
\[ v_{avg} = \frac{u + v}{2} \]  
\textit{average velocity} \\
\[ x = \frac{u + v}{2}t \]  
\textit{displacement covered in time } t \\
\[ v^2 = u^2 + 2ax \]  
\textit{timeless} \\

\[ t = \text{the time for which the body accelerates} \]
\[ a = \text{acceleration} \]
\[ u = \text{the velocity at time } t = 0, \text{ the } \textit{initial velocity} \]
\[ v = \text{the velocity after time } t, \text{ the } \textit{final velocity} \]
\[ x = \text{the displacement covered in time } t \]
Free fall is vertical (up and/or down) motion of a body where gravitational force is the only or dominant force acting upon it. (when air resistance can be ignored)

Gravitational force gives all bodies regardless of mass or shape, when air resistance can be ignored, the same acceleration.

For an object in free fall the speed would decrease by 9.8 m/s every second on the way up, at the top it would reach zero, and increase by 9.8 m/s for each successive second on the way down.

Possible lab methods to measure velocity in real situations (or acceleration) are:

**Light gates**
Device that senses when an object cuts through a beam of light. The time for which beam was broken is recorded. Knowing the length of the object, the average speed of the object through the gate can be calculated.

OR: to calculate average speed between to gates, two light gates can be used. Several light gates and a computer can be joined together to make direct calculations of velocity or acceleration.

**Strobe photography**
A strobe light gives out very brief flashes of light at fixed time intervals. If a camera is pointed at an object and the only source of light is strobe light, then the developed picture will have captured an object’s motion.

**Ticker timer**
A ticker timer can be arranged to make dots on a strip of paper at regular time intervals (typically every fiftieth of a second). If the piece of paper is attached to the object, and the object is in free fall, the dots on the strip will have recorded the distance moved by the object in known time.
\[ u = 0 \]
\[ v(1\text{sec}) = 10\text{m/s} \]
\[ v(2\text{sec}) = 20\text{m/s} \]
\[ v(3\text{sec}) = 30\text{m/s} \]
\[ v(4\text{sec}) = 40\text{m/s} \]
Solving problems using equations of motion for uniform acceleration

How far will Pinky and the Brain go in 30.0 seconds if their acceleration is 20.0 m s\(^{-2}\)?

| a = 20 m/s\(^2\) | Given |
| t = 30 s         | Given |
| u = 0 m/s        | Implicit |

\[ x = ut + \frac{1}{2}at^2 \]

\[ x = 9000 \text{ m} \]

How fast will Pinky and the Brain be going at this instant?

\[ v = u + at \quad v = 600 \text{ m/s}\]

How fast will Pinky and the Brain be going when they have traveled a total of 18000 m?

\[ v^2 = u^2 + 2ax \quad v = 850 \text{ m/s}\]
A stone is thrown vertically up from the edge of a cliff 35.0 m from the ground. The initial velocity of the stone is 8.00 m/s

(a) How high will the stone get?
(b) When will it hit the ground?
(c) What velocity will it have just before hitting the ground?
(d) What distance will the stone have covered?
(e) What is the average speed and average velocity for this motion?
(f) Make a graph to show the variation of displacement with time.
(g) Make a graph to show the variation of velocity with time.

Take the acceleration due to gravity to be 10 m/s².

\[ g = -10 \text{ ms}^{-2} \]

(a) \[ v^2 = u^2 + 2ay \Rightarrow 3.2 \text{ m from top of cliff} \]
(b) \[ y = -35 \text{ m} \quad y = ut + \left(\frac{g}{2}\right)t^2 \Rightarrow 3.56 \text{ s} \]
(c) \[ v = u + gt \quad v = -27.6 \text{ m/s} \]
(d) \[ d = 3.2m + 3.2 \text{ m} + 35 \text{ m} = 41.4 \text{ m} \]
(e) average speed = 11.6 m/s
   average velocity = -9.83 m/s
Comparison of free fall with no air resistance and with air resistance

Air resistance provides a **drag force** to objects in free fall. There is a drag force in all fluids:
- The drag force increases as the speed of the falling object increases resulting in decreasing downward acceleration until it reaches zero.
- When the drag force reaches the magnitude of the gravitational force, the falling object will stop accelerating and continue falling at a constant velocity.

This is called **terminal velocity/speed**.
A **projectile** is an object that has been given an initial velocity by some sort of short-lived force, and then moves through the air under the influence of gravity and friction. Regardless of the air resistance, the vertical and the horizontal components of the motion are independent. Their combined effects produce unique path - parabola.
Analysing projectile motion

- $a_x = 0$ in the absence of air resistance.
- $a_y = -10 \text{m/s}^2$ in the absence of air resistance.

Separation of motion into horizontal and vertical component

- The flight time is depends on vertical motion ($u_y$).
- The maximum height is on vertical motion ($u_y$).
- The horizontal component of initial velocity ($u_x$) determines range.
A ball is kicked at an angle to the horizontal. The diagram shows the position of the ball every 0.50 s.

The acceleration of free fall is $g = 10 \text{ m s}^{-2}$. Air resistance may be neglected.

(b) On the diagram above draw a line to indicate a possible path for the ball if air resistance were not negligible.
A cannon fires a projectile with a muzzle velocity of 56 ms\(^{-1}\) at an angle of inclination of 15\(^0\).

(a) What are \(u_x\) and \(u_y\)?

\[
\begin{align*}
u_x &= u \cos \theta \\
u_x &= 56 \cos 15^0 \\
u_x &= 54 \text{ m s}^{-1}
\end{align*}
\]

\[
\begin{align*}
u_y &= u \sin \theta \\
u_y &= 56 \sin 15^0 \\
u_y &= 15 \text{ m s}^{-1}
\end{align*}
\]

(b) What are the equations of motion?

\[
\begin{align*}
\Delta x &= 54t \\
y &= 15t - 5t^2
\end{align*}
\]
(c) When will the ball reach its maximum height?

At the maximum height, \( v_y = 0 \).

\[ v_y = 15 - 10t \rightarrow 0 = 15 - 10t \quad t = 1.5 \text{ s} \]

(d) How far from the muzzle will the ball be when it reaches the height of the muzzle at the end of its trajectory

From symmetry \( t_{up} = t_{down} = 1.5 \text{ s} \) so \( t = 3.0 \text{ s} \).

\[ \Delta x = 54t \rightarrow \Delta x = 54(3.0) \]

\[ \Delta x = 160 \text{ m} \]

\[ v_x = 54 \quad v_y = 15 - 10t \]

(e) Sketch the following graphs:

- \( a \) vs. \( t \), \( v_x \) vs. \( t \), \( v_y \) vs. \( t \):

  The only acceleration is \( g \) in the \( y \)-direction

  \[ v_x = 54 \text{ m/s} \], a constant over time.

  \[ v_y = 15 - 10t \]

  linear with a negative gradient and it crosses the time axis at 1.5 s.
Analysing projectile motion

A stone is thrown horizontally from the top of a vertical cliff of height 33 m as shown.

The initial horizontal velocity of the stone is 18 ms$^{-1}$ and air resistance may be assumed to be negligible.

(a) State values for the horizontal and for the vertical acceleration of the stone.

Horizontal acceleration: $a_x = 0$.
Vertical acceleration: $a_y = -10$ ms$^{-2}$.

(b) Determine the time taken for the stone to reach sea level.

• Fall time determined by the height: \[ \Delta y = u_y t - 5t^2 \rightarrow -33 = 0t - 5t^2 \]
\[ t = 2.6 \text{ s} \]

(c) Calculate the distance of the stone from the base of the cliff when it reaches sea level.

• find $\Delta x$ at $t = 2.6$ s: \[ \Delta x = u_xt \rightarrow \Delta x = 18(2.6) \]
\[ \Delta x = 47 \text{ m} \]

(d) Calculate the angle that the velocity makes with the surface of the sea.

\[ v_x = u_x = 18 \text{ ms}^{-1} \quad \tan \theta = \frac{26}{18} \]
\[ v_y = u_y - 10t = -26 \text{ ms}^{-1} \quad \theta = 55^0 \]
Analysing projectile motion

A ball is kicked at an angle to the horizontal. The diagram shows the position of the ball every 0.5 s. The acceleration of free fall is $g = 10 \text{ ms}^{-1}$. Air resistance may be neglected.

(a) Using the diagram determine, for the ball

(i) The horizontal component of the initial velocity

$$\Delta x = u_x t \rightarrow u_x = \frac{\Delta x}{t} = \frac{4 \text{ m}}{0.50 \text{ s}}$$

$$u_x = 8 \text{ m s}^{-1}$$

(ii) The vertical component of the initial velocity

$$\Delta y = u_y t - 5t^2 \rightarrow u_y = \frac{\Delta y}{t} + 5t$$

$$u_y = 24.5 \text{ m s}^{-1} \approx 25 \text{ m s}^{-1}$$

(iii) the displacement after 3.0 s. $\vec{D} = 38 \text{ m} @ 51^0$

$$\text{mag. of displacement} = \sqrt{24^2 + 30^2} = 38 \text{ m} \quad \theta = \arctan \frac{30}{24} = 51^0$$

(b) On the diagram above draw a line to indicate a possible path for the ball if air resistance were not negligible.

- New peak below and left.
- Pre-peak greater than post-peak.
Inertia is resistance an object has to a change of velocity

Mass is numerical measure of the inertia of a body (kg)

Weight is the gravitational force acting on an object. \( W = mg \)

Force is an influence on an object that causes the object to accelerate
  • 1 N is the force that causes a 1-kg object to accelerate 1 m/s\(^2\).

\( F_{\text{net}} \) (resultant force) is the vector sum of all forces acting on an object

Free Body Diagram is a sketch of a body and all forces acting on it.
Newton’s first law: An object continues in motion with constant speed in a straight line (constant velocity) or stays at rest unless acted upon by a net external force.

Object is in **translational equilibrium** if $F_{\text{net}} = 0, \ a = 0 \rightarrow$ no change in velocity

Newton’s second law: $F = \frac{\Delta p}{\Delta t}$

- $\Delta p$ is the change in momentum produced by the net force $F$ in time $\Delta t$.
- $\Delta p = \Delta (mv)$

1. velocity changes, mass doesn’t change: $\Delta p = m\Delta v \rightarrow F = ma$
   - If a net force is acting on an object of mass $m$, object will acquire acceleration
   - Direction of acceleration is direction of the net force.

2. mass changes, velocity doesn’t change: $\Delta p = v\Delta m \rightarrow F = \frac{\Delta m}{\Delta t} v$
   - $\Delta m/\Delta t$ in (kg/s)

Newton’s third law: Whenever object A exerts force on object B, object B exerts an equal in magnitude but opposite in direction force on object A
Normal/Reaction force \((F_n \text{ or } R)\) is the force which is preventing an object from falling through the surface of another body.
That’s why normal force is always perpendicular (normal) to the surfaces in contact.

when you have to draw FREE BODY DIAGRAM (object and all forces acting on it), there is a requirement to draw as many normal/reaction forces as there are points of contacts.
For example if you have a 2-D car (with two wheels) you have to draw two normal/reaction forces. Each on one wheel.
Table with two legs – the same thing.
Emu with two legs – the same thing

Friction force is the force that opposes slipping (relative motion) between two surfaces in contact; it acts parallel to surface in direction opposed to slipping. Friction depends on type and roughness of surfaces and normal force.
vertical direction:

\[ F \sin \theta + F_n = mg \]

Horizontal direction:

\[ F \cos \theta - F_{fr} = ma \]
direction perpendicular to the incline:
\[ F_{\text{net}} = ma = 0 \]
\[ F_n = mg \cos \theta \]

direction along the incline direction:
\[ F_{\text{net}} = ma \]
\[ mg \sin \theta - F_{\text{fr}} = ma \]
Solid friction

- Friction acts opposite to the intended direction of motion, and parallel to the contact surface.

- Suppose we begin to pull a crate to the right, with gradually increasing force.

- We plot the applied force, and the friction force, as functions of time:

  - During the static phase, the static friction force $F_s$ exactly matches the applied force.

  - The friction force then almost instantaneously decreases to a constant value $F_d$, called the dynamic friction force.

  - $F_s$ increases linearly until it reaches a maximum value $F_{s,\text{max}}$.

- Properties of the friction force:

  \[
  0 \leq F_s \leq F_{s,\text{max}} \quad F_d < F_{s,\text{max}} \quad F_d = \text{a constant}
  \]

  \[
  F_{fr} \leq \mu_s R \quad \text{static} \quad F_{fr} = \mu_d R \quad \text{dynamic} \quad R \equiv F_n
  \]
Describing solid friction by coefficients of friction

EXAMPLE: A piece of wood with a coin on it is raised on one end until the coin just begins to slip. The angle the wood makes with the horizontal is $\theta = 15^\circ$.

What is the coefficient of static friction?

\[
\begin{align*}
\sum F_y &= 0 \\
R - mg \cos 15^\circ &= 0
\end{align*}
\]

\[
R = mg \cos 15^\circ
\]

\[
\begin{align*}
\sum F_x &= 0 \\
F_f - mg \sin 15^\circ &= 0
\end{align*}
\]

\[
F_f = mg \sin 15^\circ
\]

\[
F_f = \mu_s R
\]

\[
mg \sin 15^\circ = \mu_s mg \cos 15^\circ
\]

\[
\mu_s = \frac{mg \sin 15^\circ}{mg \cos 15^\circ} = \tan 15^\circ = 0.268
\]

Coefficient of static friction between the metal of the coin and the wood of the plank is 0.268.
Describing solid friction by coefficients of friction

EXAMPLE: Now suppose the plank of wood is long enough so that you can lower it to the point that the coin keeps slipping, but no longer accelerates \( (v = 0) \).

If this new angle is \( 12^\circ \), what is the coefficient of dynamic friction?

\[
\begin{align*}
\sum F_y &= 0 \\
R - mg \cos 12^\circ &= 0 \\
R &= mg \cos 12^\circ \\
\sum F_x &= 0 \\
F_f - mg \sin 12^\circ &= 0 \\
F_f &= mg \sin 12^\circ \\
F_f &= \mu_d R \\
mg \sin 12^\circ &= \mu_d mg \cos 12^\circ \\
\mu_d &= \tan 12^\circ = 0.213
\end{align*}
\]

Coefficient of dynamic friction between the metal of the coin and the wood of the plank is 0.213.
Air Resistance (drag force) and Terminal Velocity

When an object moves through air or any other fluid, the fluid exerts drag force on the moving object. Unlike the friction between surfaces, however, this force depends upon the speed of the object, becoming larger as the speed increases. It also depends upon the size and the shape of the object and the density and kind of fluid.

A falling object accelerates due to the gravitational force, \( mg \), exerted on it by the earth. As the object accelerates, however, its speed increases and the drag on it becomes greater and greater until it is equal to the weight of the object. At this point, the net force on the falling object is zero, so it no longer accelerates. Its speed now remains constant; it is traveling at its terminal speed. Terminal speed occurs when the weight force (down) is equaled by the drag force (up).

Simplification of something that is pretty complicated: we can write the air resistance force (or drag force) as \( f_{\text{drag}} = bv \) for very small, slow objects, or \( f_{\text{drag}} = bv^2 \) for "human-size objects, depending on the situation. “b” is a constant.
Draw all forces that act on a parachutist. Find $\sum \vec{F}$ and acceleration for

a. parachutist that has just stepped out of the airplane.

\[ \Sigma \vec{F} = m \vec{g} \quad a = \frac{mg}{m} \]

\[ a = g \]

b. parachutist is falling at increasing speed.

\[ \Sigma \vec{F} = m \vec{g} - F_{air} \quad a = \frac{mg - F_{air}}{m} \]

\[ a < g \]

the speed is still increasing, and therefore air friction too until

c. parachutist is traveling downward with constant velocity (terminal velocity)

\[ \Sigma \vec{F} = 0 \]

\[ F_{net} = 0 \]

\[ a = 0 \]
**Linear momentum** is defined as the product of an object’s mass and its velocity:

\[ p = mv \text{ vector! (kg m/s)} \]

**Impulse** is defined as the product of the net force acting on an object and time interval of action:

\[ F\Delta t \text{ vector! (Ns)} \]

Impulse \( F\Delta t \) acting on an object will produce the change of its momentum \( \Delta p \):

\[ F\Delta t = \Delta p \quad \Delta p = mv - mu \quad \text{Ns} = \text{kg m/s} \]

Achieving the same change in momentum over a longer time requires smaller force, and over a shorter time requires greater force.

WHEN YOU TRY TO FIND CHANGE IN MOMENTUM REMEMBER TO LABEL VELOCITIES AS POSITIVE OR NEGATIVE

**The impulse of a time-varying force** is represented by the area under the force-time graph.
**Law of Conservation of Momentum:** The total momentum of a system of interacting particles is conserved - remains constant, provided there is no resultant external force. Such a system is called an “isolated system”.

\[
momentum\ of\ the\ system\ after\ collision = momentum\ of\ the\ system\ before\ collision\ for\ isolated\ system \ (p = \sum p_i) \quad \text{vector sum}
\]

REMEMBER TO DRAW A SKETCH OF THE MASSES AND VELOCITIES BEFORE AND AFTER COLLISION. LABEL VELOCITIES AS POSITIVE OR NEGATIVE.

**Elastic collision:** both momentum and kinetic/mechanical energy are conserved. That means no energy is converted into thermal energy

**Inelastic collision:** momentum is conserved but KE is not conserved.

**Perfectly inelastic collision:** the most of KE is converted into other forms of energy when objects after collision stick together.

To find how much of KE is converted into thermal energy, subtract KE of the system after collision from KE of the system before the collision.

If explosion happens in an isolated system **momentum is conserved** but KE increases (input of energy from a fuel or explosive material.)
Work is the product of the component of the force in the direction of displacement and the magnitude of the displacement. (scalar)

\[ W = F_d \cos \theta \]  

\( W = F \cos \theta \)  

(Joules)

Work done by a varying force \( F \) along the whole distance travelled is the area under the graph \( F \cos \theta \) versus distance travelled.

Energy is the ability to do work. Work changes energy.

Potential energy is stored energy.

(Change in) Gravitational potential energy \( \Delta E_p = mg \Delta h \)

Elastic potential energy = work is done by external force in stretching/compressing the spring by extension \( x \).

\[ E_{PE} = W = \frac{1}{2} k x^2 \]

Kinetic energy is the energy an object possesses due to motion \( E_K = \frac{1}{2} mv^2 \)
**Work done by applied/external force** is converted into (changes) potential energy (when net force is zero, so there is no acceleration).

**Work – Kinetic energy relationship:** work done by *net* force changes kinetic energy:

\[ W = \Delta KE = \text{final } E_k - \text{initial } E_k = \frac{1}{2} mv^2 - \frac{1}{2} mu^2 \]

the work done by *centripetal force is zero*:

\[ W_{\text{net}} = 0 \rightarrow W_{\text{net}} = \Delta KE \rightarrow \text{no change in KE} \rightarrow \text{no change in speed}; \]

centripetal force cannot change the speed, only direction.

Examples: gravitational force on the moon, magnetic force on the moving charge.
**Conservation of energy law:** Energy cannot be created or destroyed; it can only be changed from one form to another.

For the system that has only mechanical energy (ME = potential energies + kinetic energy) and there is no frictional force acting on it, so no mechanical energy is converted into thermal energy, mechanical energy is conserved

\[ ME_1 = ME_2 = ME_3 = ME_4 \quad \text{mgh}_1 + \frac{1}{2}mv_1^2 = \text{mgh}_2 + \frac{1}{2}mv_2^2 = \cdots \cdots \cdots \]

If friction cannot be neglected we have to take into account work done by friction force which doesn’t belong to the object alone but is shared with environment as thermal energy. Friction converts part of kinetic energy of the object into thermal energy. Frictional force has *dissipated* energy:

\[ ME_1 - F_{fr} d = ME_2 \quad (W_{fr} = -F_{fr} d) \]
**Power** is the rate at which work is performed or the rate at which energy is transmitted/converted.

\[
P = \frac{\Delta E}{t} = \frac{W}{t}
\]

scalar \((1 \text{ W(Watt)} = 1 \text{ J/ 1s})\)

Another way to calculate **power**

\[
P = Fv
\]

\[
P = \frac{W}{t} = \frac{Fd}{t} = F \frac{d}{t}
\]

**Efficiency** is the ratio of how much work, energy or power we get out of a system compared to how much is put in.

\[
\text{eff} = \frac{W_{\text{out}}}{W_{\text{in}}} = \frac{E_{\text{out}}}{E_{\text{in}}} = \frac{P_{\text{out}}}{P_{\text{in}}}
\]
**Centripetal acceleration** causes change in direction of velocity, but doesn’t change speed. It is always directed toward the center of the circle.

\[ a_c = \frac{v^2}{r} \]

**Centripetal force** \[ F_c = ma_c = \frac{mv^2}{r} \]

**Period** \( T \): time required for one complete revolution/circle

speed around circle of radius \( r \): \[ v = \frac{2\pi R}{T} \]