OSCILLATIONS AND WAVES
### OSCILLATIONS and WAVES

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**Oscillations**

- **Oscillations** are vibrations which repeat themselves.

**EXAMPLE:** Oscillations can be driven externally, like a pendulum in a gravitational field.

**EXAMPLE:** Oscillations can be driven internally, like a mass on a spring.
Isochronous oscillations are oscillations that repeat in the same time period no matter what amplitude changes due to damping occur.

**Periodic motion**

Periodic motion is motion repeated in equal intervals of time.

Period (T) is the time duration of the cycle.

Equilibrium position is the point where displacement (x) is zero. Displacement is a vector, and as the motion is 1-D, it is sufficient to specify the direction as being positive or negative.

Amplitude ($x_0$ or A) is the maximum value of the displacement (positive).

Frequency ($f$) of an oscillation is the number of oscillations per unit time (in Hz) $T = \frac{1}{f}$
What do all of them have in common?
Restoring force !!!! Proportional to displacement
And the same math -😊
Simple harmonic motion (SHM)

In order to perform SHM an object must have a restoring force acting on it, that is:

- The magnitude of the force (⇒ acceleration) is proportional to the displacement of the body from a fixed point.
- The direction of the force (and therefore the acceleration) is always towards that fixed point.
- Mathematically:
  \[ a \propto -x \text{ or } F \propto -x \]

- The negative sign indicates that the acceleration is directed towards equilibrium, as \( x \) is directed away from equilibrium.

- When \( x \) is max. or min. the velocity is zero, and acceleration and force are maximum in the direction opposite to displacement.
- When \( x = 0 \), object is at equilibrium position, \( a = 0 \), \( F = 0 \) and \( v \) is maximum.

Even if friction or air resistance decreases the amplitude, the period remains the same. Period is CONSTANT and does NOT depend on amplitude.
Sketching and interpreting graphs of SHM

The graph is a sine curve (but if we chose to start measuring the displacement from any other time it could just as easily be a cosine, or a negative sine, or any other sinusoidally shaped graph).

If we want to find the velocity at any particular time we simply need to find the gradient of the displacement–time graph at that time.

With a curved graph we must draw a tangent to the curve (at our chosen time) in order to find gradient.

- velocity is a slope (derivative) of displacement

- acceleration is a slope (derivative) of velocity

Since $a \propto -x$, $a$ is just a reflection of $x$
Phase and phase difference

The three graphs are all sinusoidal – they take the same shape as a sine curve. The difference between them is that the graphs all start at different points on the sine curve and continue like this. The graphs are said to have a phase difference. Phase difference can be given in term of periods, or more common we use angles. When the phase difference is 0 or $T$ then two systems are said to be oscillating in phase.
Damping: Due to the presence of resistance/friction forces on oscillations in the opposite direction to the direction of motion of the oscillating particle.

Amplitude of oscillations decreases
Friction force is a dissipative force.

"to damp" is to decrease the amplitude of an oscillation.

Decreasing the amplitude doesn’t change period.

In a damped system over a long period of time the maximum height of the bob and its maximum speed will gradually decay. The energy gradually transfers into the internal energy of the bob and the air around it. But period/frequency will remain the same ⇒ Isochronous oscillations

Energy changes in SHM
Travelling waves

Transverse waves have many similarities to longitudinal waves, but there are equally many differences. An electromagnetic wave requires no medium through which to travel, but a mechanical wave such as sound does need a medium to carry it; yet the intensity of each depends upon the square of the amplitude and the two waves use the same wave equation.

Waves are of two fundamental types:

- **mechanical waves**, which require a material medium through which to travel
- **electromagnetic waves**, which can travel through a vacuum.

Both types of wave motion can be treated analytically by equations of the same form. Modelling waves can help us to understand the properties of light, radio, sound ... even aspects of the behaviour of electrons.

**Travelling waves Characteristics:**

On reflection at fixed end the pulse is inverted. The pulse has undergone a phase change of 180° or π radians on reflection. When the end is free to move there is no phase change on reflection, and the pulse travels back on the same side that it went out.

- A wave is initiated by a vibrating object (the source) and it travels away from the source.
- The particles of the medium vibrate about their rest position at the same frequency as the source.
- The wave transfers energy from one place to another.
When a wave (energy) **propagates** through a medium, oscillations of the particles of the medium are **simple harmonic**.

Progressive waves transfer energy through a distortion that travels away from the source of distortion. There is **no net** transfer of medium.

- **Transverse waves** are waves in which the particles of the medium oscillate perpendicular to the direction in which the wave is traveling.
  - **EM waves**, Earthquake secondary waves, waves on a stringed musical instrument, waves on the rope.
    - **Useful**: microwave oven, radio.
    - **Harmful**: bridges, airplane wings, internal organs in the case of heavy machinery.

- **Longitudinal waves** are waves in which the particles of the medium vibrate parallel to the direction in which the wave is traveling.
  - **Sound waves in any medium**, shock waves in an earthquake, compression waves along a spring
vocabulary:

**Wavelength** \( \lambda \) is the shortest distance between two points that are in phase on a wave, i.e. two consecutive **crests** or two consecutive **troughs**.

**Frequency** \( f \) is the number of vibrations per second performed by the source of the waves and so is equivalent to the number of crests passing a fixed point per second.

**Period** \( T \) is the time that it takes for one complete wavelength to pass a fixed point or for a particle to undergo one complete oscillation.

**Amplitude** \( A \) is the maximum displacement of a wave from its rest position.

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*Energy* of a wave of amplitude \( A \) is proportional to the amplitude\(^2\) : 

\[ E \propto A^2 \]

---

Although the **speed** of a mechanical wave depends **only** on the medium, there is a relationship between wavelength \( \lambda \), frequency \( f \) (period \( T \)) and the speed

\[ v = \frac{\lambda}{T} = \lambda f \]

---

EM waves are waves, so: \( c = \lambda f \)

- greater \( \lambda \) smaller \( f \)
There are two types of graph that are generally used when describing waves: **displacement–distance** and **displacement–time** graphs. Such graphs are applicable to every type of wave. For example **displacement** could represent:
- the displacement of the water surface from its normal flat position for water waves
- the displacement of air molecules from their normal position for sound waves
- the value of the electric field strength vector for electromagnetic waves.
Visible light is one part of a much larger spectrum of similar waves that are all electromagnetic.

EM waves are produced/generated by accelerated charges.

All electromagnetic waves (except gamma rays) are produced when electrons undergo an energy change, even though the mechanisms might differ:
- Radio waves are emitted when electrons are accelerated in an aerial or antenna.
- Gamma rays are emitted by a nucleus or by means of other particle decays or annihilation events.

EM wave is made up of changing electric and magnetic fields.

The electric and magnetic field components of EM wave are perpendicular to each other and also perpendicular to the direction of wave propagation—hence EM waves are transverse waves.

They all travel through vacuum with the same speed—speed of light $c$:

$$c = 2.99 \ 792 \ 458 \times 10^8 \ m/s \quad c \approx 3 \times 10^8 \ m/s$$

This speed is completely independent of the frequency or the wavelength of the wave!!

EM waves are waves, so: $c = \lambda f$

greater $\lambda$ smaller $f$

As the human eye is sensitive to the electric component, the amplitude of an electromagnetic wave is usually taken as the wave’s maximum electric field strength.

Those electromagnetic waves with frequencies higher than that of visible light ionize atoms—and are thus harmful to people. Those with lower frequencies are generally believed to be safe.
The nature of electromagnetic waves

Visible light is just a tiny fraction of the complete electromagnetic spectrum.
Wavefronts propagating from a point source

Wavefront is the set of points having the same phase.

Rays – show direction in which the wave travels, show direction of transfer of energy

Rays and wavefronts are perpendicular to each other

plane waves: far away from the source spherical wavefronts become plane waves (straight lines)
The intensity of waves

The loudness of a sound wave or the brightness of a light depends on the amount of energy that is received by an observer. The energy $E$ is found to be proportional to the square of the amplitude $A$: \[ E \propto A^2 \]

Loudness is the observer’s perception of the intensity of a sound and brightness that of light; loudness and brightness are each affected by frequency.

Total energy from a point source will spread out over the surface area of a sphere. Energy per second too. This means that the intensity ($I$) at a distance ($r$) from a point source is given by the power divided by the surface area of the sphere at that radius:

\[ I = \frac{P}{4\pi r^2} \]

Inverse square law. The SI unit for intensity is W m$^{-2}$.

Energy, power and intensity are all proportional to $A^2$
Polarization

Although transverse and longitudinal waves have common properties – they reflect, refract, diffract and superpose – the difference between them can be seen by the property of polarization. Polarization of a transverse wave restricts the direction of oscillation to a plane perpendicular to the direction of propagation. Longitudinal waves, such as sound waves, do not exhibit polarization because, for these waves, the direction of oscillation is parallel to the direction of propagation.

This can be rope through a fence or EM wave through polarizing glasses.

Most naturally occurring electromagnetic waves are completely unpolarize; this means the electric field vector (and therefore the magnetic field vector perpendicular to it) vibrate in random directions but in a plane always at right angles to the direction of propagation of the wave. An ideal polarizer is polarizing filter that produces linearly polarized light from unpolarized light. It is made of crystal chains hat allows electric field to pass through only in the direction that is perpendicular to the chains. The rest is absorbed.
When a pair of Polaroids are oriented to be at $90^\circ$ to each other, or “crossed”, no light is able to pass through. The first Polaroid restricts the electric field to the direction perpendicular to the crystal chains (transmitted is electric field parallel to transmission axis); the second Polaroid has its crystals aligned in this direction and so absorbs the remaining energy. The first of the two Polaroids is called the polarizer and the second is called the analyser. In general case:

Polarized light with the electric field vector of amplitude $E_0$ is incident on an analyser. The axis of transmission of the analyser makes an angle $\theta$ with the incident light. The electric field vector $E_0$ can be resolved into two perpendicular components $E_0 \cos \theta$ and $E_0 \sin \theta$. The analyser transmits the component that is parallel to its transmission axis, which is $E_0 \cos \theta$.

Intensity is proportional to the square of the amplitude of a wave so $I_0 \propto E_0^2$ the transmitted intensity is proportional to $I \propto (E_0 \cos \theta)^2$

**Malus law:**

If linearly polarized light passes through a polarizer, the intensity of the light transmitted is given by

$$I = I_0 \cos^2 \theta$$

where $\theta$ is the angle between the polarization direction of the light and the transmission axis of the polarizer.

- A substance is termed **optically active** if the plane of polarized light rotates as it passes through the substance. A sugar solution is an example of such a substance. So is quartz.
Brewster’s law/angle

If $\theta_{\text{refl}} + \theta_{\text{refr}} = 90^0$ then the reflected ray will be completely plane-polarized.

$$\theta_{\text{inc}} + \theta_{\text{refr}} = 90^0$$

The particular angle of incidence at which this total polarization occurs is called **Brewster’s angle**
Law of Reflection

The incident and reflected wavefronts.

Angle of reflection is equal to angle of incidence.

\[ \angle i = \angle r. \]

(the angles are measured to the normal to the barrier).

All waves, including light, sound, water obey this relationship, the law of reflection.
When a wave passes from one medium to another, its velocity changes. The change in speed results in a change in direction of propagation of the refracted wave.

Visualization of refraction

As a toy car rolls from a hardwood floor onto carpet, it changes direction because the wheel that hits the carpet first is slowed down first.
The incident and refracted wavefronts.

frequency is determined by the source so it doesn’t change. Only wavelength changes. Wavelength of the same wave is smaller in the medium with smaller speed.
A mathematical law which will tell us exactly HOW MUCH the direction has changed is called **SNELL'S LAW**.

Although it can be derived by using little geometry and algebra, it was introduced as experimental law for light in 1621.

For a given pair of media, the ratio

\[
\frac{\sin \theta_1}{\sin \theta_2} = \frac{v_1}{v_2}
\]

is constant for the given frequency. The Snell’s law is of course valid for all types of waves.
The speed of light inside matter

- The speed of light \( c = 300,000,000 \text{ m/s} = 3 \times 10^8 \text{ m/s} \)
- In any other medium such as water or glass, light travels at a lower speed.

- INDEX OF REFRACTION, \( n \), of the medium is the ratio of the speed of light in a vacuum, \( c \), and the speed of light, \( v \), in that medium:

\[
 n = \frac{c}{v}
\]

no units

As \( c \) is greater than \( v \) for all media, \( n \) will always be \( > 1 \).

greater \( n \) – smaller speed of light in the medium.

As the speed of light in air is almost equal to \( c \), \( n_{air} \sim 1 \)
SNELL'S LAW for EM waves

\[
\frac{\sin \theta_1}{\sin \theta_2} = \frac{v_1}{v_2}
\]

Can be written in another form for refraction of light only.

\[
\frac{v_1}{v_2} = \frac{c}{n_1} = \frac{n_2}{n_1} \rightarrow n_1 \sin \theta_1 = n_2 \sin \theta_2
\]

greater n ⇒ smaller speed of light ⇒ stronger refraction ⇒ smaller angle
Angle of refraction is greater than angle of incidence. As the angle of incidence increases, so does angle of refraction. The intensity of refracted light decreases, intensity of reflected light increases until angle of incidence is such that angle of refraction is 90°. 

Critical angle: \( \theta_c \) - angle of incidence for which angle of refraction is 90° 

When the incident angle is greater than \( \theta_c \), the refracted ray disappears and the incident ray is totally reflected back.
Critical angle: $\theta_c$ - angle of incidence for which angle of refraction is $90^0$

$$n_1 \sin \theta_1 = n_2 \sin \theta_2 \rightarrow n_1 \sin \theta_c = n_2 \sin 90^0 \rightarrow$$

$$\sin \theta_c = \frac{n_2}{n_1}$$
**Chromatic dispersion** is phenomenon in which the index of refraction depends on wavelength/frequency, so the speed of light through a material varies slightly with the frequency of the light and each $\lambda$ is refracted at a slightly different angle.

The longer $\lambda$, the smaller index of refraction.

$$n_{\text{red}} < n_{\text{blue}}$$, red light is refracted less than blue light

Dispersion is the phenomenon which gives separation of colours in prism/rainbow and undesirable chromatic aberration in lenses.

**Diffraction** is the spreading of a wave into a region behind an obstruction (into a region of geometrical shadow).

Diffraction effects are more obvious when wavelength of the wave is similar in size to aperture/obstacle or bigger.

*remember:* big $\lambda$ (compared to aperture or obstacle), **big diffraction effects**
Interference is the addition (superposition) of two or more waves overlapping that results in a new wave pattern.

**Principle of superposition:** When two or more waves overlap, the resultant displacement at any point and at any instant is the vector sum of the displacements of the individual waves at that point: \( y = y_1 + y_2 \)

\( PD \) = path difference is the difference in distances traveled by waves from two sources to a point \( P \): \( PD = d_2 - d_1 \)

Two coherent waves traveling along two different paths to the same point will:

* **interfere constructively** if the difference in distance traveled is equal to a whole number of wavelengths:
  \[ PD = n \lambda \quad n = 0, \pm 1, \pm 2, \pm 3, \ldots \]

* **interfere destructively** if the difference in distance traveled is equal to a half number of wavelengths:
  \[ PD = (n + \frac{1}{2}) \lambda \quad n = 0, \pm 1, \pm 2, \pm 3, \ldots \]
Young’s double-slit interference experiment

Laser light is used as source $S_0$.

Two waves from two slits will always start the journey with equal phase (we say they are coherent), so interference pattern – distribution of constructive and destructive interference depends on their path difference.
The geometry of Young’s double slit experiment

- $S_1$ and $S_2$ two coherent, monochromatic point sources.
- $D$ - distance from the sources to the screen (meters)
- $d$ - distance between the slits (in parts of mm)
  - The waves from the two sources will be in phase at $Q$ and there will be a bright fringe here.
  - What happens at $P$ distance $s$ from $Q$?
    - $S_1P = AP$, so
    - $S_2A$ is path difference $= d \sin \theta$

- The condition for a bright fringe at $P$ is constructive interference.
  - Path difference $d \sin \theta$ must be:
    - $d \sin \theta = n\lambda$
    - $n = 0, \pm 1, \pm 2, \pm 3, \ldots$

- The condition for a dark fringe at $P$ is destructive interference.
  - Path difference $d \sin \theta$ must be:
    - $d \sin \theta = (n + \frac{1}{2})\lambda$
    - $n = 0, \pm 1, \pm 2, \pm 3, \ldots$

We want to find fringe spacing $s$: let’s assume first bright fringe, then $d \sin \theta = \lambda$

- $S_1P$ is effectively perpendicular to $S_2P$, therefore

\[
\tan \theta = \frac{s}{D} \quad \text{&} \quad \sin \theta = \frac{\lambda}{d}
\]

\[
\frac{s}{d} = \frac{\lambda}{D} \quad \rightarrow \quad s = \frac{\lambda D}{d}
\]

This gives the separation of successive bright fringes (or bands of loud sound for a sound experiment).

- Young actually use this expression to measure the wavelength of the light he used and it is a method still used today.
Standing Waves

Standing waves are the result of the interference of two identical waves with the same frequency and the same amplitude traveling in opposite direction. It can happen ONLY at certain frequencies that are multiple of basic one.

The frequencies at which standing waves are produced are called **natural frequencies** or **resonant frequencies** of the string or pipe or...

the lowest freq. standing wave is called FUNDAMENTAL or the FIRST HARMONICS
The higher freq. standing waves are called HARMONICS (second, third...) or OVERTONES
Standing waves on string fixed at both ends

A string has a number of frequencies at which it will naturally vibrate. These natural frequencies are known as the harmonics of string. The natural frequency at which a string vibrates depends upon the tension of the string, the mass per unit length and the length of the string. With a stringed musical instrument each end of a string is fixed, meaning there will be a node at either end.

A node is a point where the standing wave has minimal amplitude  
A antinode is a point where the standing wave has maximal amplitude  
Distance between two nodes is $\lambda/2$  

1. harmonic: $L = \frac{\lambda}{2} \rightarrow \lambda = 2L \rightarrow f_1 = \frac{v}{\lambda_1} = \frac{v}{2L}$  
2. harmonic: $L = \lambda \rightarrow \lambda = L \rightarrow f_2 = \frac{v}{\lambda_2} = \frac{v}{L} = 2 \frac{v}{2L}$  
3. harmonic: $L = \frac{3\lambda}{2} \rightarrow \lambda = \frac{2}{3}L \rightarrow f_3 = \frac{v}{\lambda_3} = \frac{3v}{2L} = 3 \frac{v}{2L}$  
4. harmonic: $L = 2\lambda \rightarrow \lambda = \frac{L}{2} \rightarrow f_4 = \frac{v}{\lambda_4} = \frac{2v}{L} = 4 \frac{v}{2L}$  

$f_n = n \frac{v}{2L} = n f_1$
Many musical instruments depend on the musician in some way moving air through the instrument.  
▪ This includes brass and woodwind instruments, as well as instruments like pipe organs.  
▪ All instruments like this can be divided into two categories, open ended or closed ended.  
▪ A “pipe” can be any tube, even if it has been bent into different shapes or has holes cut into it.

Remember that it is actually air that is doing the vibrating as a wave here.

![Diagram of air vibrations in a pipe](image)

The air at the closed end of the pipe must be a node (not moving), since the air is not free to move there and must be able to be reflected back.

There must also be an antinode where the opening is, since that is where there is maximum movement of the air.

The frequencies of sounds made by these two types of instruments are different because of the different ways that air will move at a closed or open end of the pipe.
Boundary conditions – closed pipes

• We can also set up standing waves in pipes.
• In the case of pipes, longitudinal waves are created and these waves are reflected from the ends of the pipe.
• Consider a closed pipe of length \( L \) which gets its wave energy from a mouthpiece on the left side.

Why must the mouthpiece end be an antinode?
Why must the closed end be a node?

1. harmonic: \( L = \frac{\lambda_1}{4} \rightarrow \lambda_1 = 4L \rightarrow f_1 = \frac{v}{\lambda_1} = \frac{v}{4L} \)

3. harmonic: \( L = 3 \frac{\lambda_3}{4} \rightarrow \lambda_3 = \frac{4L}{3} \rightarrow f_3 = \frac{v}{\lambda_3} = 3 \frac{v}{4L} = 3f_1 \)

5. harmonic: \( L = 5 \frac{\lambda_5}{4} \rightarrow \lambda_5 = \frac{4L}{5} \rightarrow f_5 = \frac{v}{\lambda_5} = 5 \frac{v}{4L} = 5f_1 \)

Source: molecules can be displaced by the large amount here. Air can’t move.
**Boundary conditions – open pipes**

- In an open-ended pipe you there is an antinode at the open end because the medium *can* vibrate there (and, of course, antinode at the mouthpiece).

1. harmonic: \( L = \frac{\lambda_1}{2} \rightarrow \lambda_1 = 2L \) \( \rightarrow f_1 = \frac{v}{\lambda_1} = \frac{v}{2L} \)

2. harmonic: \( L = \lambda_2 \rightarrow \lambda_2 = L \) \( \rightarrow f_2 = \frac{v}{\lambda_2} = 2 \frac{v}{2L} = 2f_1 \)

3. harmonic: \( L = 3\frac{\lambda_3}{2} \rightarrow \lambda_3 = \frac{2L}{3} \) \( \rightarrow f_3 = \frac{v}{\lambda_3} = 3 \frac{v}{2L} = 3f_1 \)

- The IBO requires you to be able to make sketches of string and pipe harmonics (both open and closed) and find wavelengths and frequencies.