Maria travels to school either by walking or by bicycle. The probability she cycles to school is 0.75.

If she walks, the probability that she is late for school is 0.1.
If she cycles, the probability that she is late for school is 0.05.

1a. Complete the tree diagram below, showing the appropriate probabilities.

1b. Find the probability that Maria is late for school.
**Markscheme**

P(late) = 0.25 \times 0.1 + 0.75 \times 0.05 \quad (AI)(AI)(MI)

**Note:** Award (AI)(AI) for two correct products from their diagram and award (MI) for addition of their two products.

= 0.0625 \left( \frac{1}{16}, 6.25\% \right) \quad (AI)(AI) \quad (C3)

[3 marks]

**Examiners report**

Part (a) of this question was very well answered with many candidates gaining the maximum marks. Many candidates were less successful in part (b) and it seemed as if many of them either gained 3 marks or 0 marks. This shows that students who knew how to approach part (b) were also able to correctly substitute in the formula they used and reach the correct answer. Very few of those students lost the last mark for wrong rounding.

In a research project on the relation between the gender of 150 science students at college and their degree subject, the following set of data is collected.

<table>
<thead>
<tr>
<th>Gender</th>
<th>Biology</th>
<th>Physics</th>
<th>Chemistry</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>40</td>
<td>16</td>
<td>35</td>
</tr>
<tr>
<td>Female</td>
<td>15</td>
<td>24</td>
<td>20</td>
</tr>
</tbody>
</table>

2a. Find the probability that a student chosen at random is male. \[2 \text{ marks}\]

**Markscheme**

= \frac{91}{150} \left(0.607, 60.6\%, 60.7\%\right) \quad (AI)(AI) \quad (C2)

**Note:** Award (AI) for numerator, (AI) for denominator.

[2 marks]

**Examiners report**

Parts (a) and (b) were well answered with many candidates gaining 4 marks there. The conditional probability in part (c) proved to be more challenging. Nearly all candidates attempted this question showing that time was not a factor in this paper. Many candidates gave their answers as incorrectly rounded decimals, which incurred an accuracy penalty and prevented them from gaining the maximum marks.

2b. Find the probability that a student chosen at random is either male or studies Chemistry. \[2 \text{ marks}\]
A group of 30 students were asked about their favourite topping for toast.

18 liked peanut butter (A)
10 liked jam (B)
6 liked neither

Show this information on the Venn diagram below. [2 marks]
3b. Find the number of students who like both peanut butter and jam. 

**Markscheme**

\[ 18 + 10 + 6 = 30 \quad (M1) \]
\[ = 4 \quad (A1) \quad (C2) \]

[2 marks]

**Examiners report**

The first two parts of this question were well answered with most candidates completing the Venn diagram correctly and finding the number in the intersection. The final part, requiring a conditional probability to be found, proved more difficult as many candidates tried to use the formula, when all that was required was to look at the values in the Venn diagram. Follow through marks were awarded in part (c) for values correctly used from parts (a) and (b).

3c. Find the probability that a randomly chosen student from the group likes peanut butter, given that they like jam.

**Markscheme**

\[ P(A|B) = \frac{4}{10} = \frac{2}{5} = 0.4 = 40\% \quad (A1)(ft)(A1) \quad (C2) \]

**Note:** Award (A1)(ft) for their numerator from part (b), (A1) for denominator.

[2 marks]
Examiners report
The first two parts of this question were well answered with most candidates completing the Venn diagram correctly and finding the number in the intersection. The final part, requiring a conditional probability to be found, proved more difficult as many candidates tried to use the formula, when all that was required was to look at the values in the Venn diagram. Follow through marks were awarded in part (c) for values correctly used from parts (a) and (b).

A bag contains 7 red discs and 4 blue discs. Ju Shen chooses a disc at random from the bag and removes it. Ramón then chooses a disc from those left in the bag.

4a. Write down the probability that
(i) Ju Shen chooses a red disc from the bag;
(ii) Ramón chooses a blue disc from the bag, given that Ju Shen has chosen a red disc;
(iii) Ju Shen chooses a red disc and Ramón chooses a blue disc from the bag.

Markscheme
(i) \( \frac{7}{11} \) (0.636, 63.6\%) (0.636363\ldots) (A1) (C1)
(ii) \( \frac{4}{10} \) (0.4, 40\%) (A1) (C1)
(iii) \( \frac{28}{110} \) (0.255, 25.5\%) \( 0.254545\ldots \) (A1)(ft) (C1)

Note: Follow through from the product of their answers to parts (a) (i) and (ii).

[3 marks]

Examiners report
The vast majority of candidates were able to pick up the first two marks by confidently identifying the number of favourable outcomes/total number of outcomes. Difficulties arose however when combining events and only the more able candidates were able to progress successfully with the remainder of the question. As usual in this type of question, there was an abundance of incorrect answers greater than 1 given.

4b. Find the probability that Ju Shen and Ramón choose different coloured discs from the bag.

Markscheme
\[
\frac{28}{110} + \left( \frac{4}{11} \times \frac{7}{10} \right) \quad \text{OR} \quad 2 \times \frac{28}{110} \quad (M1)(M1)
\]
Notes: Award (M1) for using their \( \frac{28}{110} \) as part of a combined probability expression. (M1) for either adding \( \frac{4}{11} \times \frac{7}{10} \) or for multiplying by 2.

\[
= \frac{56}{110} \quad (0.509, 50.9\%) \quad (A1)(ft) \quad (C3)
\]
Note: Follow through applies from their answer to part (a) (iii) and only when their answer is between 0 and 1.

[3 marks]
Examiners report

The vast majority of candidates were able to pick up the first two marks by confidently identifying the number of favourable outcomes/total number of outcomes. Difficulties arose however when combining events and only the more able candidates were able to progress successfully with the remainder of the question. As usual in this type of question, there was an abundance of incorrect answers greater than 1 given.

A survey was carried out in a group of 200 people. They were asked whether they smoke or not. The collected information was organized in the following table.

<table>
<thead>
<tr>
<th></th>
<th>Smoker</th>
<th>Non-smoker</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>60</td>
<td>40</td>
</tr>
<tr>
<td>Female</td>
<td>30</td>
<td>70</td>
</tr>
</tbody>
</table>

One person from this group is chosen at random.

5a. Write down the probability that this person is a smoker. [2 marks]

Markscheme

\[
\frac{60}{200} (0.45, 45 \%) \quad (A1)(A1) \quad (C2)
\]

Note: Award (A1) for numerator, (A1) for denominator.

5b. Write down the probability that this person is male given that they are a smoker. [2 marks]

Markscheme

\[
\frac{60}{90} (0.6, 0.667, 66.6 \%, 66.6\ldots \%, 66.7 \%) \quad (A1)(A1)(ft) \quad (C2)
\]

Notes: Award (A1) for numerator, (A1)(ft) for denominator, follow through from their numerator in part (a). Last mark is lost if answer is not a probability.

5c. Find the probability that this person is a smoker or is male. [2 marks]

Examiners report

This question was generally well answered by many of the candidates. Many found the conditional probability in part b) easier compared to previous sessions, since they were able to write it down directly from the table. A number of candidates found the final part difficult with a significant number unable to use the combined events probability formula correctly.
Markscheme

\[
\frac{100}{200} \quad \frac{100}{200} \quad \frac{60}{200} \quad (MI)
\]

Note: Award (MI) for correct substitution in the combined events formula. Follow through from their answer to part (a).

\[
\frac{100}{200} (0.65, 65\%) \quad (AI)(ft)
\]

OR

\[
\frac{60}{200} + \frac{40}{200} + \frac{30}{200} \quad (MI)
\]

Note: Award (MI) for adding the correct fractions.

\[
\frac{100}{200} (0.65, 65\%) \quad (AI)
\]

OR

\[
1 - \frac{70}{200} \quad (MI)
\]

Note: Award (MI) for subtraction of correct fraction from 1.

\[
\frac{100}{200} (0.65, 65\%) \quad (AI) \quad (C2)
\]

[2 marks]

Examiners report

This question was generally well answered by many of the candidates. Many found the conditional probability in part b) easier compared to previous sessions, since they were able to write it down directly from the table. A number of candidates found the final part difficult with a significant number unable to use the combined events probability formula correctly.

Part A

100 students are asked what they had for breakfast on a particular morning. There were three choices: cereal (X), bread (Y) and fruit (Z). It is found that

- 10 students had all three
- 17 students had bread and fruit only
- 15 students had cereal and fruit only
- 12 students had cereal and bread only
- 13 students had only bread
- 8 students had only cereal
- 9 students had only fruit

6a. Represent this information on a Venn diagram. [4 marks]
Markscheme

(A1) for rectangle and three intersecting circles

(A1) for 10, (A1) for 8, 13 and 9, (A1) for 12, 15 and 17  (A4)

[4 marks]

Examiners report

This question was in general well done. Candidates began the paper well by drawing the Venn diagram correctly. Some students omitted the rectangle (universal set) around the three circles. There were quite a few errors in (c) as some students forgot to convert their answers to percentages. Also describing in words what the students in $X \cap Y'$ had for breakfast seemed to be difficult for the majority of the candidates. Some misread what $Y$ was and even more missed the complement sign. However, the main problem in answering this question seemed to be the lack of knowledge in the relationship between set theory and logic (use of "and" and "or"). Combining probabilities caused problems to many. Common wrong answers were $\frac{10}{100} \times \frac{10}{100}$ or $\frac{10}{100} + \frac{9}{99}$.

6b. Find the number of students who had none of the three choices for breakfast.  

[2 marks]

Markscheme

$100 - (9 + 12 + 13 + 15 + 10 + 17 + 8) = 16$  (M1)(A1)(R)(G2)

Note: Follow through from their diagram.

[2 marks]

Examiners report

This question was in general well done. Candidates began the paper well by drawing the Venn diagram correctly. Some students omitted the rectangle (universal set) around the three circles. There were quite a few errors in (c) as some students forgot to convert their answers to percentages. Also describing in words what the students in $X \cap Y'$ had for breakfast seemed to be difficult for the majority of the candidates. Some misread what $Y$ was and even more missed the complement sign. However, the main problem in answering this question seemed to be the lack of knowledge in the relationship between set theory and logic (use of "and" and "or"). Combining probabilities caused problems to many. Common wrong answers were $\frac{10}{100} \times \frac{10}{100}$ or $\frac{10}{100} + \frac{9}{99}$.

6c. Write down the percentage of students who had fruit for breakfast.  

[2 marks]
**Examiners report**

This question was in general well done. Candidates began the paper well by drawing the Venn diagram correctly. Some students omitted the rectangle (universal set) around the three circles. There were quite a few errors in (c) as some students forgot to convert their answers to percentages. Also describing in words what the students in $X \cap Y'$ had for breakfast seemed to be difficult for the majority of the candidates. Some misread what $Y$ was and even more missed the complement sign. However, the main problem in answering this question seemed to be the lack of knowledge in the relationship between set theory and logic (use of “and” and “or”). Combining probabilities caused problems to many. Common wrong answers were $\frac{10}{100} \times \frac{10}{100}$, $\frac{10}{100}$ and $\frac{9}{99}$.

**Markscheme**

\[
\frac{51}{100} \ (0.51) \quad (A1)(\text{ft})
\]

\[
= 51\% \quad (A1)(\text{ft})(G2)
\]

**Note:** Follow through from their diagram.

[2 marks]

6d. Describe in words what the students in the set $X \cap Y'$ had for breakfast.

**Markscheme**

Note: The following statements are correct. Please note that the connectives are important. It is not the same (had cereal) and (not bread) and (had cereal) or (not bread). The parentheses are not needed but are there to facilitate the understanding of the propositions.

(had cereal) and (did not have bread)
(had cereal only) or (had cereal and fruit only)
(had either cereal or (fruit and cereal)) and (did not have bread) \( (A1)(AI) \)

Notes: If the statements are correct but the connectives are wrong then award at most (A1)(A0). For the statement (had only cereal) and (cereal and fruit) award (A1)(A0). For the statement had cereal and fruit award (A0)(A0).

[2 marks]

6e. Find the probability that a student had at least two of the three choices for breakfast.

**Markscheme**

\[
\frac{54}{100} \ (0.54, 54\%) \quad (A1)(\text{ft})(A1)(\text{ft})(G2)
\]

**Note:** Award (A1)(ft) for numerator, follow through from their diagram, (A1)(ft) for denominator. Follow through from total or denominator used in part (c).

[2 marks]
Examiners report

This question was in general well done. Candidates began the paper well by drawing the Venn diagram correctly. Some students omitted the rectangle (universal set) around the three circles. There were quite a few errors in (c) as some students forgot to convert their answers to percentages. Also describing in words what the students in $X \cap Y'$ had for breakfast seemed to be difficult for the majority of the candidates. Some misread what $Y$ was and even more missed the complement sign. However, the main problem in answering this question seemed to be the lack of knowledge in the relationship between set theory and logic (use of "and" and "or"). Combining probabilities caused problems to many. Common wrong answers were $\frac{10}{100} \times \frac{10}{100} \times \frac{9}{99} + \frac{9}{99}$.

6f. Two students are chosen at random. Find the probability that both students had all three choices for breakfast. 

Markscheme

\[
\frac{10}{100} \times \frac{9}{99} = \frac{1}{110}(0.00909, 0.909 \%) \quad (A1)(ft)(M1)(A1)(ft)(G2)
\]

Notes: Award (A1)(ft) for their correct fractions, (M1) for multiplying two fractions, (A1)(ft) for their correct answer. Answer 0.009 with no working receives no marks. Follow through from denominator in parts (c) and (e) and from their diagram.

[3 marks]

Examiners report

This question was in general well done. Candidates began the paper well by drawing the Venn diagram correctly. Some students omitted the rectangle (universal set) around the three circles. There were quite a few errors in (c) as some students forgot to convert their answers to percentages. Also describing in words what the students in $X \cap Y'$ had for breakfast seemed to be difficult for the majority of the candidates. Some misread what $Y$ was and even more missed the complement sign. However, the main problem in answering this question seemed to be the lack of knowledge in the relationship between set theory and logic (use of "and" and "or"). Combining probabilities caused problems to many. Common wrong answers were $\frac{10}{100} \times \frac{10}{100} \times \frac{9}{99}$.

Part B

The same 100 students are also asked how many meals on average they have per day. The data collected is organized in the following table.

<table>
<thead>
<tr>
<th>3 or fewer meals per day</th>
<th>4 or 5 meals per day</th>
<th>More than 5 meals per day</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>15</td>
<td>25</td>
<td>15</td>
</tr>
<tr>
<td>Female</td>
<td>12</td>
<td>20</td>
<td>13</td>
</tr>
<tr>
<td>Total</td>
<td>27</td>
<td>45</td>
<td>28</td>
</tr>
</tbody>
</table>

A $\chi^2$ test is carried out at the 5 % level of significance.

6g. Write down the null hypothesis, $H_0$, for this test. 

Markscheme

$H_0$: The (average) number of meals per day a student has and gender are independent \quad (AI)

Note: For “independent” accept “not associated” but do not accept “not related” or “not correlated”.

[1 mark]
Examiners report

In general this part question was well answered. The major concerns of the examining team were the following:

- In (f) many students wrote down the expected values table (from the GDC) and highlighted the correct expected value, 12.6. As this is a "show that" question the use of the GDC is not expected and therefore no marks are awarded for this working. Instead it is expected the use of the formula for the expected value with the correct substitutions.
- In (e) surprisingly many candidates found the $\chi^2$ value through the use of the formula. Unfortunately this led to some incorrect answers and also to a bad use of time. The question clearly says "use your graphic display calculator" and it is worth 2 marks therefore a student should not spend more than 2 minutes to answer this part question. Time management is essential in this type of examinations and the IB rule is one minute – one mark.

6h. Write down the number of degrees of freedom for this test.  

[1 mark]

Markscheme

2 (A1)  

[1 mark]

Examiners report

In general this part question was well answered. The major concerns of the examining team were the following:

- In (f) many students wrote down the expected values table (from the GDC) and highlighted the correct expected value, 12.6. As this is a "show that" question the use of the GDC is not expected and therefore no marks are awarded for this working. Instead it is expected the use of the formula for the expected value with the correct substitutions.
- In (e) surprisingly many candidates found the $\chi^2$ value through the use of the formula. Unfortunately this led to some incorrect answers and also to a bad use of time. The question clearly says "use your graphic display calculator" and it is worth 2 marks therefore a student should not spend more than 2 minutes to answer this part question. Time management is essential in this type of examinations and the IB rule is one minute – one mark.

6i. Write down the critical value for this test.  

[1 mark]

Markscheme

5.99 (accept 5.991) (A1)(ft)  

Note: Follow through from their part (b).  

[1 mark]

Examiners report

In general this part question was well answered. The major concerns of the examining team were the following:

- In (f) many students wrote down the expected values table (from the GDC) and highlighted the correct expected value, 12.6. As this is a "show that" question the use of the GDC is not expected and therefore no marks are awarded for this working. Instead it is expected the use of the formula for the expected value with the correct substitutions.
- In (e) surprisingly many candidates found the $\chi^2$ value through the use of the formula. Unfortunately this led to some incorrect answers and also to a bad use of time. The question clearly says "use your graphic display calculator" and it is worth 2 marks therefore a student should not spend more than 2 minutes to answer this part question. Time management is essential in this type of examinations and the IB rule is one minute – one mark.

6j. Show that the expected number of females that have more than 5 meals per day is 13, correct to the nearest integer.  

[2 marks]
6k. Use your graphic display calculator to find the $\chi^2_{\text{calc}}$ for this data.  

**Markscheme**

0.0321 \ (G2) 

**Note:** For 0.032 award (G1)(G1)(AP). For 0.03 with no working award (G0). 

**[2 marks]**

**Examiners report**

In general this part question was well answered. The major concerns of the examining team were the following:

- In (f) many students wrote down the expected values table (from the GDC) and highlighted the correct expected value, 12.6. As this is a "show that" question the use of the GDC is not expected and therefore no marks are awarded for this working. Instead it is expected the use of the formula for the expected value with the correct substitutions.
- In (e) surprisingly many candidates found the $\chi^2_{\text{calc}}$ through the use of the formula. Unfortunately this led to some incorrect answers and also to a bad use of time. The question clearly says "use your graphic display calculator" and it is worth 2 marks therefore a student should not spend more than 2 minutes to answer this part question. Time management is essential in this type of examinations and the IB rule is one minute – one mark.

6l. Decide whether $H_0$ must be accepted. Justify your answer. 

**Markscheme**

0.0321 < 5.99 or 0.984 > 0.05 \ (R1)  

accept $H_0$ \ (AI)(ft) 

**Note:** If reason is incorrect both marks are lost, do not award (R0)(AI). 

**[2 marks]**
Examiners report

In general this part question was well answered. The major concerns of the examining team were the following:

- In (f) many students wrote down the expected values table (from the GDC) and highlighted the correct expected value, 12.6. As this is a "show that" question the use of the GDC is not expected and therefore no marks are awarded for this working. Instead it is expected the use of the formula for the expected value with the correct substitutions.
- In (e) surprisingly many candidates found the \( \chi^2 \) value through the use of the formula. Unfortunately this led to some incorrect answers and also to a bad use of time. The question clearly says "use your graphic display calculator" and it is worth 2 marks therefore a student should not spend more than 2 minutes to answer this part question. Time management is essential in this type of examinations and the IB rule is one minute – one mark.

Pam has collected data from a group of 400 IB Diploma students about the Mathematics course they studied and the language in which they were examined (English, Spanish or French). The summary of her data is given below.

<table>
<thead>
<tr>
<th>Mathematics HL</th>
<th>Mathematics SL</th>
<th>Mathematical Studies SL</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>English</td>
<td>50</td>
<td>70</td>
<td>200</td>
</tr>
<tr>
<td>Spanish</td>
<td>30</td>
<td>50</td>
<td>110</td>
</tr>
<tr>
<td>French</td>
<td>20</td>
<td>30</td>
<td>90</td>
</tr>
<tr>
<td>Total</td>
<td>100</td>
<td>150</td>
<td>400</td>
</tr>
</tbody>
</table>

7a. A student is chosen at random from the group. Find the probability that the student

(i) studied Mathematics HL;
(ii) was examined in French;
(iii) studied Mathematics HL and was examined in French;
(iv) did not study Mathematics SL and was not examined in English;
(v) studied Mathematical Studies SL given that the student was examined in Spanish.

Markscheme

(i) \( \frac{100}{400} (\frac{1}{4}, 0.25, 25\%) \) \((AI)\)

(ii) \( \frac{30}{400} (\frac{3}{40}, 0.225, 22.5\%) \) \((AI)\)

(iii) \( \frac{20}{400} (\frac{1}{20}, 0.05, 5\%) \) \((AI)(AI)\)

**Note:** Award \((AI)\) for numerator, \((AI)\) for denominator.

(iv) \( \frac{120}{400} (\frac{3}{10}, 0.3, 30\%) \) \((AI)(AI)\)

**Note:** Award \((AI)\) for numerator, \((AI)\) for denominator.

(v) \( \frac{30}{110} (\frac{3}{11}, 0.273, 27.3\%) \) \((0.272727\ldots) \) \((AI)(AI)\)

**Note:** Award \((AI)\) for numerator, \((AI)\) for denominator. Accept 0.27, do not accept 0.272, do not accept 0.3. [8 marks]
**Examiners report**

The simple probabilities beginning this question were successfully attempted by the great majority. Most errors in the latter parts occurred due to candidates trying to use the algebraic form of laws of probability, rather than by interpreting the contingency table. Probability questions in this course are, in the main, contextual and the reliance of formulas is not always beneficial to the candidates. Only the best candidates realized the significance of part (b) as a link to the chi-squared test.

This was well attempted by the majority, the weakness being the sole reliance of the calculator to calculate expected value. However, there still remains confusion between critical and $p$-values as the basis for accepting the null hypothesis.

---

**7b.** Pam believes that the Mathematics course a student chooses is independent of the language in which the student is examined. **[2 marks]**

Using your answers to parts (a) (i), (ii) and (iii) above, state whether there is any evidence for Pam’s belief. Give a reason for your answer.

**Markscheme**

\[ \frac{1}{20} \neq \frac{1}{4} \times \frac{9}{40} \quad (R1)(ft) \]

*Note:* The fractions must be used as part of the reason. Follow through from (a)(i), (a)(ii) and (a)(iii).

Pam is not correct. **(A1)(ft)**

*Notes:* Do not award **(R0)(A1)**. Accept the events are not independent (dependent).

**[2 marks]**

---

**Examiners report**

The simple probabilities beginning this question were successfully attempted by the great majority. Most errors in the latter parts occurred due to candidates trying to use the algebraic form of laws of probability, rather than by interpreting the contingency table. Probability questions in this course are, in the main, contextual and the reliance of formulas is not always beneficial to the candidates. Only the best candidates realized the significance of part (b) as a link to the chi-squared test.

This was well attempted by the majority, the weakness being the sole reliance of the calculator to calculate expected value. However, there still remains confusion between critical and $p$-values as the basis for accepting the null hypothesis.

---

**7c.** Pam decides to test her belief using a Chi-squared test at the 5% level of significance. **[3 marks]**

(i) State the null hypothesis for this test.

(ii) Show that the expected number of Mathematical Studies SL students who took the examination in Spanish is 41.3, correct to 3 significant figures.

**Markscheme**

(i) The mathematics course and language of examination are independent. **(A1)**

*Notes:* Accept “There is no association between Mathematics course and language”. Do not accept “not related”, “not correlated”, “not influenced”.

(ii) \[ \frac{110}{400} \times \frac{150}{400} \times 400 \left( = \frac{110 \times 150}{400} \right) \quad (M1) \]

\[ = 41.25 \quad (AI) \]

\[ = 41.3 \quad (AG) \]

*Note:* 41.25 and 41.3 must be seen to award final (A1).

**[3 marks]**
Examiners report
The simple probabilities beginning this question were successfully attempted by the great majority. Most errors in the latter parts occurred due to candidates trying to use the algebraic form of laws of probability, rather than by interpreting the contingency table. Probability questions in this course are, in the main, contextual and the reliance of formulas is not always beneficial to the candidates. Only the best candidates realized the significance of part (b) as a link to the chi-squared test.

This was well attempted by the majority, the weakness being the sole reliance of the calculator to calculate expected value. However, there still remains confusion between critical and $p$-values as the basis for accepting the null hypothesis.

7d. Write down [4 marks]
(i) the Chi-squared calculated value;
(ii) the number of degrees of freedom;
(iii) the Chi-squared critical value.

Markscheme
(i) $7.67$ (7.67003. . .) (G2)
Note: Accept 7.7, do not accept 8 or 7.6. Award (G1) if formula with all nine terms seen but their answer is not one of those above.
(ii) 4 (G1)
(iii) $9.488$ (A1)(ft)
Notes: Accept 9.49 or 9.5, do not accept 9.4 or 9.

Examiners report
The simple probabilities beginning this question were successfully attempted by the great majority. Most errors in the latter parts occurred due to candidates trying to use the algebraic form of laws of probability, rather than by interpreting the contingency table. Probability questions in this course are, in the main, contextual and the reliance of formulas is not always beneficial to the candidates. Only the best candidates realized the significance of part (b) as a link to the chi-squared test.

This was well attempted by the majority, the weakness being the sole reliance of the calculator to calculate expected value. However, there still remains confusion between critical and $p$-values as the basis for accepting the null hypothesis.

7e. State, giving a reason, whether there is sufficient evidence at the 5% level of significance that Pam’s belief is correct. [2 marks]

Markscheme
$7.67 < 9.488$ (R1)
OR
$p = 0.104\ldots, p > 0.05$ (R1)
Accept (Do not reject) $H_0$ (Pam’s belief is correct) (A1)(ft)
Notes: Follow through from part (d). Do not award (R0)(A1).
Examiners report
The simple probabilities beginning this question were successfully attempted by the great majority. Most errors in the latter parts occurred due to candidates trying to use the algebraic form of laws of probability, rather than by interpreting the contingency table. Probability questions in this course are, in the main, contextual and the reliance of formulas is not always beneficial to the candidates. Only the best candidates realized the significance of part (b) as a link to the chi-squared test.

This was well attempted by the majority, the weakness being the sole reliance of the calculator to calculate expected value. However, there still remains confusion between critical and p-values as the basis for accepting the null hypothesis.

Given the set \( A = \{x - 4 \leq x \leq 2, x \text{ is an integer}\} \).

8a. List the elements of the set \( A \). \([1 \text{ mark}]\)

**Markscheme**
- 4, −3, −2, −1, 0, 1, 2 \((AI)\) \((C1)\)

**Note:** Award \((AI)\) for correct numbers, do not penalise if braces, brackets or parentheses seen.

\([1 \text{ mark}]\)

8b. A number is chosen at random from set \( A \). Write down the probability that the number chosen is a negative integer. \([2 \text{ marks}]\)

**Markscheme**
\(\frac{4}{7} (0.571, 57.1\%) \ (AI)(ft)(AI)(ft) \ (C2)\)

**Notes:** Award \((AI)(ft)\) for numerator, \((AI)(ft)\) for denominator. Follow through from part (a).

**Note:** There is no further penalty in parts (c) and (d) for use of denominator consistent with that in part (b).

\([2 \text{ marks}]\)

Examiners report
Most candidates were able to answer part a correctly, although many did not realise that 0 is an integer. The award of follow through marks was common in parts b, c and d.

8c. A number is chosen at random from set \( A \). Write down the probability that the number chosen is a positive even integer. \([1 \text{ mark}]\)

**Markscheme**
\(\frac{1}{7} (0.143, 14.3\%) \ (AI)(ft) \ (C1)\)

**Note:** Follow through from part (a).

\([1 \text{ mark}]\)
Examiners report
Most candidates were able to answer part a correctly, although many did not realise that 0 is an integer. The award of follow through marks was common in parts b, c and d.

8d. A number is chosen at random from set $A$. Write down the probability that the number chosen is an odd integer less than $-1$. [2 marks]

Markscheme
\[
\frac{1}{7} \times (0.143, 14.3\%) \quad (A1)(ft)(A1)(ft) \quad (C2)
\]
Note: Award $(A1)(ft)$ for numerator, $(A1)(ft)$ for denominator. Follow through from part (a).

[2 marks]

Examiners report
Most candidates were able to answer part a correctly, although many did not realise that 0 is an integer. The award of follow through marks was common in parts b, c and d.

The probability that it rains today is 0.4. If it rains today, the probability that it will rain tomorrow is 0.8. If it does not rain today, the probability that it will rain tomorrow is 0.7.

9a. Complete the tree diagram below. [3 marks]
9b. Calculate the probability of rain tomorrow.  

\[ 0.4 \times 0.8 + 0.6 \times 0.7 = 0.74 \]  

Examiners report  
Part a of this question was well answered, however part b caused many problems. Candidates did not seem to know how to find the probability of the combined events.

One day the numbers of customers at three cafés, “Alan’s Diner” (A), “Sarah’s Snackbar” (S) and “Pete’s Eats” (P), were recorded and are given below.

- 17 were customers of Pete’s Eats only
- 27 were customers of Sarah’s Snackbar only
- 15 were customers of Alan’s Diner only
- 10 were customers of Pete’s Eats and Sarah’s Snackbar but not Alan’s Diner
- 8 were customers of Pete’s Eats and Alan’s Diner but not Sarah’s Snackbar

Draw a Venn Diagram, using sets labelled A, S and P, that shows this information.
**Markscheme**

(A1) for rectangle and three labelled intersecting circles
(A1) for 15, 27 and 17
(A1) for 10 and 8 (A3)

[3 marks]

**Examiners report**

**Part A**

This part was successfully attempted by the great majority. A common mistake was the failure to intersect all three sets.

---

10b.

There were 48 customers of Pete’s Eats that day. Calculate the number of people who were customers of all three cafés.  

**Markscheme**

48 – (8 + 10 + 17) or equivalent  (MI)

= 13  (A1)(R)(G2)

[2 marks]

**Examiners report**

**Part A**

This part was successfully attempted by the great majority. A common mistake was the failure to intersect all three sets.

---

10c.

There were 50 customers of Sarah’s Snackbar that day. Calculate the total number of people who were customers of Alan’s Diner.  

[3 marks]
**Markscheme**

50 – (27 + 10 + 13) \((M1)\)

Note: Award \((M1)\) for working seen.

\[
= 0 \quad (AI)
\]

number of elements in \(A = 36 \quad (AI)(ft)(G3)\)

Note: Follow through from (b).

[3 marks]

**Examiners report**

Part A

This part was successfully attempted by the great majority. A common mistake was the failure to intersect all three sets.

---

10d. Write down the number of customers of Alan’s Diner that were also customers of Pete’s Eats. \([1\text{ mark}]\)

**Markscheme**

21 \((AI)(ft)\)

Note: Follow through from (b) even if no working seen.

[1 mark]

**Examiners report**

Part A

This part was successfully attempted by the great majority. A common mistake was the failure to intersect all three sets.

---

10e. Find \(n[(S \cup P) \cap A']\). \([2\text{ marks}]\)

**Markscheme**

54 \((MI)(AI)(ft)(G2)\)

Note: Award \((MI)\) for 17, 10, 27 seen. Follow through from (a).

[2 marks]

**Examiners report**

Part A

This part was successfully attempted by the great majority. A common mistake was the failure to intersect all three sets.

A surprising number seemed unfamiliar with set notation in (e) and thus did not attempt this part.
Some of the customers in each café were given survey forms to complete to find out if they were satisfied with the standard of service they received.

<table>
<thead>
<tr>
<th></th>
<th>Pete’s Eats</th>
<th>Alan’s Diner</th>
<th>Sarah’s Snackbar</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Satisfied</td>
<td>26</td>
<td>8</td>
<td>16</td>
<td>40</td>
</tr>
<tr>
<td>Dissatisfied</td>
<td>16</td>
<td>8</td>
<td>16</td>
<td>40</td>
</tr>
<tr>
<td>Total</td>
<td>42</td>
<td>28</td>
<td>50</td>
<td>120</td>
</tr>
</tbody>
</table>

10f. One of the survey forms was chosen at random, find the probability that the form showed “Dissatisfied”;

**Markscheme**

\[
\frac{16}{120} \quad (\frac{2}{3}, 0.333, 33.3\%) \quad (AI)(AI)(G2)
\]

Note: Award (AI) for numerator, (AI) for denominator.

[2 marks]

**Examiners report**

Part B

The work on probability also proved accessible to the great majority with a large number of candidates attaining full marks. Most errors occurred due to candidates trying to use the algebraic form of laws of probability, rather than by interpreting the contingency table.

10g. One of the survey forms was chosen at random, find the probability that the form showed “Satisfied” and was completed at Sarah’s Snackbar;

**Markscheme**

\[
\frac{16}{120} \quad (\frac{4}{15}, 0.283, 28.3\%) \quad (AI)(AI)(G2)
\]

Note: Award (AI) for numerator, (AI) for denominator.

[2 marks]

**Examiners report**

Part B

The work on probability also proved accessible to the great majority with a large number of candidates attaining full marks. Most errors occurred due to candidates trying to use the algebraic form of laws of probability, rather than by interpreting the contingency table.

10h. One of the survey forms was chosen at random, find the probability that the form showed “Dissatisfied”, given that it was completed at Alan’s Diner.

**Markscheme**

\[
\frac{8}{28} \quad (\frac{2}{7}, 0.286, 28.6\%) \quad (AI)(AI)(G2)
\]

Note: Award (AI) for numerator, (AI) for denominator.

[2 marks]
A $\chi^2$ test at the 5% significance level was carried out to determine whether there was any difference in the level of customer satisfaction in each of the cafés. Write down the null hypothesis, $H_0$, for the $\chi^2$ test.

**Markscheme**

customer satisfaction is independent of café (A1)

*Note: Accept “customer satisfaction is not associated with the café”.*

[1 mark]

A $\chi^2$ test at the 5% significance level was carried out to determine whether there was any difference in the level of customer satisfaction in each of the cafés. Write down the number of degrees of freedom for the test.

**Markscheme**

2 (A1)

[1 mark]

A $\chi^2$ test at the 5% significance level was carried out to determine whether there was any difference in the level of customer satisfaction in each of the cafés.

Using your graphic display calculator, find $\chi^2_{calc}$. 

**Examiners report**

Part B

The work on probability also proved accessible to the great majority with a large number of candidates attaining full marks. Most errors occurred due to candidates trying to use the algebraic form of laws of probability, rather than by interpreting the contingency table.

The chi-squared test was well done by the great majority, however, it was clear that a number of centres do not teach this subject, since there were a number of scripts which either were left blank or showed no understanding in the responses seen.
Examiners report

Part B

The work on probability also proved accessible to the great majority with a large number of candidates attaining full marks. Most errors occurred due to candidates trying to use the algebraic form of laws of probability, rather than by interpreting the contingency table.

The chi-squared test was well done by the great majority, however, it was clear that a number of centres do not teach this subject, since there were a number of scripts which either were left blank or showed no understanding in the responses seen.

A \( \chi^2 \) test at the 5% significance level was carried out to determine whether there was any difference in the level of customer satisfaction in each of the cafés.  

State, giving a reason, the conclusion to the test.

Markscheme

since \( \chi^2_{calc} < \chi^2_{crit} \) accept (or Do not reject) \( H_0 \)  

\((R1)(A1)(ft)\)

Note: Follow through from their value in (e).

OR

Accept (or Do not reject) \( H_0 \) as \( p \)-value (0.686) > 0.05  

\((R1)(A1)(ft)\)

Notes: Do not award \((A1)(R0)\). Award the \((R1)\) for comparison of appropriate values.

[2 marks]

Examiners report

Part B

The work on probability also proved accessible to the great majority with a large number of candidates attaining full marks. Most errors occurred due to candidates trying to use the algebraic form of laws of probability, rather than by interpreting the contingency table.

The chi-squared test was well done by the great majority, however, it was clear that a number of centres do not teach this subject, since there were a number of scripts which either were left blank or showed no understanding in the responses seen.

A survey was carried out at an international airport. A number of travellers were interviewed and asked for their flight destinations. The results are shown in the table below.

<table>
<thead>
<tr>
<th>Destination</th>
<th>America</th>
<th>Africa</th>
<th>Asia</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of males</td>
<td>45</td>
<td>62</td>
<td>37</td>
</tr>
<tr>
<td>Number of females</td>
<td>35</td>
<td>46</td>
<td>25</td>
</tr>
</tbody>
</table>

One traveller is to be chosen at random from all those interviewed.

Find the probability that this traveller was going to Africa.
11b. One female traveller is to be chosen at random from all those interviewed. [2 marks]

Find the probability that this female traveller was going to Asia.

Markscheme

\( \frac{25}{106} \) (0.236, 23.6%) \( (A1)(A1) \) (C2)

Note: Award (A1) for numerator, (A1) for denominator.

[2 marks]

Examiners report

A reasonably well attempted question with parts (a) and (c) proving to provide many correct answers. A correct answer for part (b) however proved to be a little more elusive as, despite a correct numerator of 25 seen on many scripts, the total sample space was not reduced and a denominator of 250 lost the final mark in this part of the question. On a minority of scripts candidates simply wrote down decimal answers. Where these were correct, both marks for each part were earned. However, incorrect answers earned no marks – candidates would be well advised to at least write down the fraction answer first so that any part marks can be awarded. A case in question here was a predominance of incorrect answers of 0.10 or 10% for part (b). This, on its own earns no marks whereas 25/250 earned A1, A0.

11c. One traveller is to be chosen at random from those not going to America. [2 marks]

Find the probability that the chosen traveller is female.

Markscheme

\( \frac{71}{170} \) (0.418, 41.8%) \( (A1)(A1) \) (C2)

Note: Award (A1) for numerator, (A1) for denominator.

[2 marks]

Examiners report

A reasonably well attempted question with parts (a) and (c) proving to provide many correct answers. A correct answer for part (b) however proved to be a little more elusive as, despite a correct numerator of 25 seen on many scripts, the total sample space was not reduced and a denominator of 250 lost the final mark in this part of the question. On a minority of scripts candidates simply wrote down decimal answers. Where these were correct, both marks for each part were earned. However, incorrect answers earned no marks – candidates would be well advised to at least write down the fraction answer first so that any part marks can be awarded. A case in question here was a predominance of incorrect answers of 0.10 or 10% for part (b). This, on its own earns no marks whereas 25/250 earned A1, A0.
Let \( P(A) = 0.5 \), \( P(B) = 0.6 \) and \( P(A \cup B) = 0.8 \).

12a. Find \( P(A \cap B) \).

**Markscheme**

0.8 = 0.5 + 0.6 - \( P(A \cap B) \) \( (MI) \)

\( P(A \cap B) = 0.3 \) \( (A1) \) \( (C2) \)

**Note:** Award \( (MI) \) for correct substitution, \( (A1) \) for correct answer.

12b. Find \( P(A | B) \).

**Markscheme**

\( P(A | B) = \frac{0.3}{0.6} \) \( (MI) \)

= 0.5 \( (A1)(ft) \) \( (C2) \)

**Note:** Award \( (MI) \) for correct substitution in conditional probability formula. Follow through from their answer to part (a), provided probability is not greater than one.

12c. Decide whether \( A \) and \( B \) are independent events. Give a reason for your answer.

\( P(A \cap B) = P(A) \times P(B) \)

\( P(A | B) = P(A) \)
Phoebe chooses a biscuit from a blue tin on a shelf. The tin contains one chocolate biscuit and four plain biscuits. She eats the biscuit and chooses another one from the tin. The tree diagram below represents the situation with the four possible outcomes where $A$ stands for chocolate biscuit and $B$ for plain biscuit.

### 13a. Write down the value of $a$. [1 mark]

**Markscheme**

$a = 0 \left( \frac{2}{5} \right)$ (AI) [1 mark]

**Examiners report**

This question was well handled by most of the candidates except for (c)(ii) in which they had to find a conditional probability. Some candidates did not copy the second tree diagram in the answer sheets and instead wrote their answers in the exam booklet thus losing the 3 marks allocated to part (b).

### 13b. Write down the value of $b$. [2 marks]

**Markscheme**

$b = \frac{3}{5} (0.75, 75\%)$ (A2)(G2) [2 marks]

**Examiners report**

This question was well handled by most of the candidates except for (c)(ii) in which they had to find a conditional probability. Some candidates did not copy the second tree diagram in the answer sheets and instead wrote their answers in the exam booklet thus losing the 3 marks allocated to part (b).

### 13c. Find the probability that both biscuits are plain. [3 marks]
Markscheme

\[
\frac{4}{5} \times \frac{2}{4} \quad (M1)(A1)
\]

\[
\frac{15}{25} \left( \frac{2}{5}, 0.6, 60\% \right) \quad (A1)(R)(G2)
\]

Note: Award \( (M1) \) for multiplying two probabilities, \( (A1) \) for using their probabilities, \( (A1) \) for answer.

[3 marks]

Examiners report

This question was well handled by most of the candidates except for (c)(ii) in which they had to find a conditional probability. Some candidates did not copy the second tree diagram in the answer sheets and instead wrote their answers in the exam booklet thus losing the 3 marks allocated to part (b).

On another shelf there are two tins, one red and one green. The red tin contains three chocolate biscuits and seven plain biscuits and the green tin contains one chocolate biscuit and four plain biscuits. Andrew randomly chooses either the red or the green tin and randomly selects a biscuit.

13d. Copy and complete the tree diagram below.

[3 marks]

Markscheme

\[
\begin{aligned}
\text{Red} & \quad A \\
& \quad B \\
\text{Green} & \quad A \\
& \quad B
\end{aligned}
\]

\[
\begin{aligned}
\frac{1}{2} & \quad \text{Red} \\
\frac{1}{2} & \quad \text{Green}
\end{aligned}
\]

\[
\begin{aligned}
\frac{3}{10} & \quad \text{A} \\
\frac{7}{10} & \quad \text{B}
\end{aligned}
\]

\[
\begin{aligned}
\frac{1}{5} & \quad \text{A} \\
\frac{4}{5} & \quad \text{B}
\end{aligned}
\]

Note: Award \( (A1) \) for each pair.

[3 marks]
**Examiners report**

This question was well handled by most of the candidates except for (c)(ii) in which they had to find a conditional probability. Some candidates did not copy the second tree diagram in the answer sheets and instead wrote their answers in the exam booklet thus losing the 3 marks allocated to part (b).

13e. Find the probability that he chooses a chocolate biscuit.  

**Markscheme**

\[
\frac{1}{2} \times \frac{3}{10} + \frac{1}{2} \times \frac{1}{5} \quad (M1)(M1)
\]

\[
= \frac{5}{20} \left( \frac{1}{4}, 0.25, 25\% \right) \quad (A1)(R)(G2)
\]

**Note:** Award (M1) for two products seen with numbers from the problem, (M1) for adding two products. Follow through from their tree diagram.

13f. Find the probability that he chooses a biscuit from the red tin given that it is a chocolate biscuit.  

**Markscheme**

\[
\frac{\frac{1}{2} \times 3}{\frac{1}{2}} \quad (M1)(A1)
\]

\[
= \frac{3}{5} \quad (A1)(R)(G2)
\]

**Note:** Award (M1) for substituted conditional probability formula, (A1) for correct substitution.

Follow through from their part (b) and part (c) (i).

**Examiners report**

This question was well handled by most of the candidates except for (c)(ii) in which they had to find a conditional probability. Some candidates did not copy the second tree diagram in the answer sheets and instead wrote their answers in the exam booklet thus losing the 3 marks allocated to part (b).
The probability that it will snow tomorrow is 0.3.
If it snows tomorrow the probability that Chuck will be late for school is 0.8.
If it does not snow tomorrow the probability that Chuck will be late for school is 0.1.

14a. Complete the tree diagram below.

\[ \begin{array}{c}
\text{Snow} \\
0.3 \\
\text{Not late} \\
\text{Late} \\
\text{No snow} \\
0.2 \\
\text{Late} \\
\text{Not late} \\
\end{array} \]

\[ \text{Late} \quad 0.8 \]

\[ \text{Not late} \quad 0.2 \]

Markscheme

\[(A1)(A1)(A1) \quad (C3)\]

Note: Award (A1) for each correct pair.

Examiners report

This question was answered well.

14b. Find the probability that it does not snow tomorrow and Chuck is late for school.

\[ 0.7 \times 0.1 = 0.07 = 7\% \]

Markscheme

\[ 0.7 \times 0.1 = 0.07 = 7\% \quad (A1)(\Omega) \quad (C1) \]

Examiners report

This question was answered well.
14c. Find the probability that Chuck is late for school. [2 marks]

**Markscheme**

\[ 0.3 \times 0.8 + 0.07 \quad (M1) \]
\[ = 0.31 (\frac{31}{100}, \text{31\%}) \quad (A1)(ft) \]

**Note:** In (b) and (c) follow through from sensible answers only i.e. not a probability greater than one. \( (C2) \)

[2 marks]

**Examiners report**

A few students were unable to do part (c).

---

15a. A class consists of students studying Spanish or French or both. Fifteen students study Spanish and twelve study French. The probability that a student studies French given that she studies Spanish is \( \frac{7}{15} \).

Draw a Venn diagram in the space below to illustrate this information. [3 marks]

**Markscheme**

\[ A \rightarrow \text{Spanish}, \quad B \rightarrow \text{French} \]

\[ U \]

\[ \begin{align*}
S &\quad 8 \\
7 \quad 5 \\
\end{align*} \]

\( (A1)(A1)(A1) \quad (C3) \)

**Note:** Award \( (A1) \) for a labeled Venn diagram with appropriate sets. \( (A1) \) for 7, \( (A1) \) for 8 and 5.

[3 marks]

**Examiners report**

Part (a) was done well.

---

15b. Find the probability that a student studies Spanish given that she studies one language only. [3 marks]
16a. Find the probability that the two dice show the same number.

**Markscheme**

\[ \frac{4}{24} = \frac{1}{6} (0.167, 16.7\%) \] 

(A1)(A1) (C2)

**Note:** Award (A1) for numerator, (A1) for denominator.

[2 marks]

**Examiners report**

The diagram caused some difficulty for some candidates, however the majority of candidates were successful.

16b. Find the probability that the difference between the two numbers shown on the dice is 1.

**Markscheme**

\[ \frac{4}{24} \] 

(A1)(A1) (C2)

[2 marks]
Markscheme
\[ \frac{7}{24} \text{ (0.292, 29.2\%)} \quad (A1)(A1)(ft) \quad (C2) \]

Note: Award $(A1)(ft)$ from the denominator used in (a).

[2 marks]

Examiners report
The diagram caused some difficulty for some candidates, however the majority of candidates were successful in (a).
The term “difference” was well understood by the candidature.

16c. Find the probability that the number shown on the four-sided die is greater than the number shown on the six-sided die, given that the difference between the two numbers is 1.

Markscheme
\[ \frac{3}{7} \text{ (0.429, 42.9\%)} \quad (A1)(A1)(ft) \quad (C2) \]

Note: Award $(A1)$ for numerator $(A1)(ft)$ for denominator, $(ft)$ from their numerator in (b).

[2 marks]

Examiners report
The diagram caused some difficulty for some candidates, however the majority of candidates were successful in (a).

Sharon and Lisa share a flat. Sharon cooks dinner three nights out of ten. If Sharon does not cook dinner, then Lisa does. If Sharon cooks dinner the probability that they have pasta is 0.75. If Lisa cooks dinner the probability that they have pasta is 0.12.

17a. Copy and complete the tree diagram to represent this information.

[3 marks]
Examiners report

17b. Find the probability that Lisa cooks dinner and they do not have pasta.

\[ 0.7 \times 0.88 = 0.616 \ (\text{or} \ 61.6\%) \] (\text{MI}(A1)(A0)(G2)}

Note: Award (MI) for multiplying the correct probabilities.

[2 marks]

Examiners report

The tree diagram was quite well answered by many students, but sometimes it was missing on many papers. It seemed they had it on their examination paper because the subsequent questions were answered accurately. Conditional probability was of great difficulty to many candidates.
Markscheme

0.3 \times 0.25 + 0.7 \times 0.88 \quad (M1)(M1)

Notes: Award (M1) for a relevant two-factor product, could be $S \times NP$ OR $L \times NP$.
Award (M1) for summing 2 two-factor products.

P = 0.691 \left( \frac{691}{1000}, 69.1 \text{ }\% \right) \quad (A1)(ft)(G2)

Notes: (ft) from their answer to (b).

[3 marks]

Examiners report

The tree diagram was quite well answered by many students, but sometimes it was missing on many papers. It seemed they had it on their examination paper because the subsequent questions were answered accurately. Conditional probability was of great difficulty to many candidates.

Given that they do not have pasta, find the probability that Lisa cooked dinner.

[3 marks]

Markscheme

\frac{0.616}{0.691} \quad (M1)(A1)

Note: Award (M1) for substituted conditional probability formula, (A1) for correct substitution.

P = 0.891 \left( \frac{891}{980}, 89.1 \text{ }\% \right) \quad (A1)(ft)(G2)

[3 marks]

Examiners report

The tree diagram was quite well answered by many students, but sometimes it was missing on many papers. It seemed they had it on their examination paper because the subsequent questions were answered accurately. Conditional probability was of great difficulty to many candidates.

A survey was carried out in a year 12 class. The pupils were asked which pop groups they like out of the Rockers (R), the Salseros (S), and the Blues (B). The results are shown in the following diagram.

\[ U \]

\[ R \quad 2 \quad 2x \quad 0 \]
\[ S \]
\[ x \quad 3 \quad 5 \]
\[ 4 \quad B \]

17d. Write down $n(R \cap S \cap B)$.

[1 mark]
Markscheme

3 \((A1)\)  
[1 mark]

Examiners report

This question was well handled although part (d) proved too difficult for many candidates and demonstrated, overall, a poor level of understanding of basic set notation. Students generally had the algebraic skills required to solve for \(x\) in part (e)(ii).

17f. Find \(n(R')\).  

Markscheme

For 5, 4, 7 (0) seen with no extra values \((A1)\)  
16 \((A1)(G2)\)  
[2 marks]

Examiners report

This question was well handled although part (d) proved too difficult for many candidates and demonstrated, overall, a poor level of understanding of basic set notation. Students generally had the algebraic skills required to solve for \(x\) in part (e)(ii).

17g. Describe which groups the pupils in the set \(S \cap B\) like.  

Markscheme

They like (both) the Salseros \((S)\) and they like the Bluers \((B)\) \((A1)(A1)\)  

Note: Award \((A1)\) for “and”, \((A1)\) for the correct groups.  
[2 marks]

Examiners report

This question was well handled although part (d) proved too difficult for many candidates and demonstrated, overall, a poor level of understanding of basic set notation. Students generally had the algebraic skills required to solve for \(x\) in part (e)(ii).

17h. Use set notation to describe the group of pupils who like the Rockers and the Bluers but do not like the Salseros.  

Markscheme

\(R \cap B \cap S'\) \((A1)(A1)\)  

Note: Award \((A1)\) for \(R \cap B\), \((A1)\) for \(\cap S'\)  
[2 marks]
17i. There are 33 pupils in the class. Find $x$.  

**Markscheme**

21 + 3\(x\) = 33 \((M1)\)

\(x = 4\) \((A1)(G2)\)

\([2 \text{ marks}]\)

Examiners report

This question was well handled although part (d) proved too difficult for many candidates and demonstrated, overall, a poor level of understanding of basic set notation. Students generally had the algebraic skills required to solve for $x$ in part (e)(ii).

17j. There are 33 pupils in the class. Find the number of pupils who like the Rockers. 

**Markscheme**

17 \((A1)(\text{ft})\)

\([1 \text{ mark}]\)

A survey of 100 families was carried out, asking about the pets they own. The results are given below.

- 56 owned dogs \((S)\)
- 38 owned cats \((Q)\)
- 22 owned birds \((R)\)
- 16 owned dogs and cats, but not birds
- 8 owned birds and cats, but not dogs
- 3 owned dogs and birds, but not cats
- 4 owned all three types of pets

18a. Draw a Venn diagram to represent this information.  

\([5 \text{ marks}]\)
**Markscheme**

Note: Award (A1) for rectangle (U not required), (A1) for 3 intersecting circles, (A1) for 4 in central intersection, (A1) for 16, 3, 8 and (A1) for 33, 10, 7 (ft) if subtraction is carried out, or for S(56), Q(38) and R(22) seen by the circles.

[5 marks]

**Examiners report**
Most candidates began the paper well by correctly drawing the Venn diagram and answering parts (b) and (c) correctly.

18b. Find the number of families who own no pets. [2 marks]

**Markscheme**

100 – 81 (MI)

19 (A1)(ft)(G2)

Note: Award (MI) for subtracting their total from 100.

[2 marks]

**Examiners report**
Most candidates began the paper well by correctly drawing the Venn diagram and answering parts (b) and (c) correctly.

18c. Find the percentage of families that own exactly one pet. [3 marks]
Markscheme

\[ 33 + 10 + 7 \quad (M1) \]

Note: Award \((M1)\) for adding their values from (a).

\[ \left( \frac{50}{100} \right) \times 100 \% \quad (A1)(ft) \]

\[ 50 \% (50) \quad (A1)(ft)(G3) \]

[3 marks]

Examiners report

Most candidates began the paper well by correctly drawing the Venn diagram and answering parts (b) and (c) correctly.

18d. A family is chosen at random. Find the probability that they own a cat, given that they own a bird. \([2 marks]\)

Markscheme

\[ P \text{ (own a cat given they own a bird)} = \frac{\frac{12}{22}}{\frac{5}{11}} \quad (A1)(ft)(A1)(ft) \]

Note: Award \((A1)(ft)\) for the numerator, \((A1)(ft)\) for the denominator.

[2 marks]

Examiners report

Conditional probability has proved difficult for many candidates; only a very small part of the candidates scored full marks for this part.

For events \(A\) and \(B\), the probabilities are \(P(A) = \frac{4}{13}\) and \(P(B) = \frac{5}{13}\).

19a. If events \(A\) and \(B\) are mutually exclusive, write down the value of \(P(A \cap B)\). \([1 mark]\)

Markscheme

\[ P(A \cap B) = 0 \quad (A1) \quad (CI) \]

[1 mark]

Examiners report

This question proved to be difficult with many candidates unaware of the significance of mutually exclusive events in probability. A significant number gave the answer to (b) as the answer to (a).

19b. If events \(A\) and \(B\) are independent, find the value of \(P(A \cap B)\). \([2 marks]\)
### Markscheme

\[ P(A \cap B) = P(A) \times P(B) \]
\[ = \frac{4}{13} \times \frac{5}{13} \quad (MI) \]

**Note:** Award (MI) for product of two fractions, decimals or percentages.

\[ P(A \cap B) = \frac{20}{169} (= 0.118) \quad (AI) \quad (C2) \]

[2 marks]

### Examiners report

This question proved to be difficult with many candidates unaware of the significance of mutually exclusive events in probability. A significant number gave the answer to (b) as the answer to (a).

#### 19c.

If \( P(A \cup B) = \frac{7}{13} \), find the value of \( P(A \cap B) \).

[3 marks]

### Markscheme

\[ \frac{7}{13} = \frac{4}{13} + \frac{5}{13} - P(A \cap B) \quad (MI)(MI) \]

**Notes:** Award (MI) for \( \frac{4}{13} + \frac{5}{13} \) seen, (MI) for subtraction of \( \frac{7}{13} \) shown.

OR

Award (MI) for Venn diagram with 2 intersecting circles, (AI) for correct probabilities in diagram.

\[ P(A \cap B) = \frac{2}{13} (= 0.154) \quad (AI) \quad (C3) \]

[3 marks]

### Examiners report

This question proved to be difficult with many candidates unaware of the significance of mutually exclusive events in probability. This part proved to be difficult for some but most of the candidates who used the formula were able to achieve full marks. Very few candidates used Venn diagrams to answer this question.

---

A **weighted** die has 2 red faces, 3 green faces and 1 black face. When the die is thrown, the black face is three times as likely to appear on top as one of the other five faces. The other five faces have equal probability of appearing on top.

The following table gives the probabilities.

<table>
<thead>
<tr>
<th></th>
<th>Red 1</th>
<th>Red 2</th>
<th>Green 1</th>
<th>Green 2</th>
<th>Green 3</th>
<th>Black</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \frac{1}{8} )</td>
<td>( \frac{1}{8} )</td>
<td>( \frac{1}{8} )</td>
<td>( \frac{m}{8} )</td>
<td>( \frac{1}{8} )</td>
<td>( \frac{n}{8} )</td>
</tr>
</tbody>
</table>

20a.

Find the value of

(i) \( m \);

(ii) \( n \).
### Markscheme

(i) \( m = 1 \) \((A1)\)

(ii) \( n = 3 \) \((A1) \) \((C2)\)

Note: Award \((A0)(A1)(ft)\) for \( m = \frac{1}{4}, n = \frac{3}{7}\).

Award \((A0)(A1)(ft)\) for \( m = 3, n = 1\).

[2 marks]

### Examiners report

The answers 1/8 and 3/8 were provided by many rather than 1 and 3. The conditional probability question was correctly answered more often when the formula was used. A common incorrect answer to part (c) was 3/8 \( \times \) 2/7.

---

20b. The die is thrown once. 

Given that the face on top is not red, find the probability that it is black. 

### Markscheme

\[
P(B/\bar{R}) = \frac{\frac{2}{6}}{\frac{4}{6}} = \frac{3}{6} = \frac{1}{2} (50\%, 0.5) \quad (M1)(A1)(ft) \quad (C2)
\]

Note: Award \((M1)\) for correctly substituted conditional probability formula or for 6 seen as part of denominator.

[2 marks]

### Examiners report

The answers 1/8 and 3/8 were provided by many rather than 1 and 3. The conditional probability question was correctly answered more often when the formula was used. A common incorrect answer to part (c) was 3/8 \( \times \) 2/7.

\[
P(B/B') = \frac{\frac{3}{4} \times \frac{3}{4}}{\frac{3}{4}} = \frac{9}{64} (0.141)
\]

---

20c. The die is now thrown twice. 

Calculate the probability that black appears on top both times.

### Markscheme

\[
P(B,B) = \frac{\frac{3}{4}}{\frac{3}{4}} = \frac{9}{64} (0.141) \quad (M1)(A1)(ft) \quad (C2)
\]

Note: Award \((M1)\) for product of two correct fractions, decimals or percentages.

(ft) from their answer to part (a) (ii).

[2 marks]

### Examiners report

The answers 1/8 and 3/8 were provided by many rather than 1 and 3. The conditional probability question was correctly answered more often when the formula was used. A common incorrect answer to part (c) was 3/8 \( \times \) 2/7.