Practice Problems - PROJECTILE MOTION

Problem 1: A shotput is thrown. For the each of the indicated positions of the shotput along its trajectory, draw and label the following vectors: the x-component of the velocity, the y-component of the velocity, and the acceleration. Explain why you drew the vectors as you did.

Problem 2: A rock is thrown with an initial vertical velocity component of 30 m/s and an initial horizontal velocity component of 40 m/s.
   a. What will these velocity components be one second after the rock reaches the top of its path?
   b. Assuming the launch and landing heights are the same, how long will the rock be in the air?
   c. Assuming the launch and landing heights are the same, how far will the rock land from where it was thrown?

Problem 3: Two tennis ball launchers shoot balls at the same time, angle and initial speed from different floors of a tall building. The balls land in the street below. Ignore air resistance.

   a. Which ball will have the greater acceleration while in flight? Explain your reasoning.
   b. Which ball will hit farther from the base of the building? Explain your reasoning.
   c. Which ball will reach a greater maximum height? Explain your reasoning.
   d. Which ball will be going faster just before hitting the street? Explain your reasoning.
   e. How could you adjust only the angle of the upper launcher so that the ball hits in the same place as the ball from the lower launcher? Explain your reasoning.
   f. How could you adjust only the angle of the lower launcher so that the ball hits in the same place as the ball from the upper launcher? Explain your reasoning.

Problem 4: Balls A and B are launched from different heights. The reach the same maximum height at exactly the same point in space.

   a. Which ball has a greater initial vertical component of velocity? Explain.
   b. Which ball has a greater initial horizontal component of velocity? Explain.
   c. Which ball has the larger launch angle? Explain.
   d. Which ball has greater acceleration while in flight? Explain.
   e. Which ball will land farther from the launchers? Explain.
   f. Which ball takes longer to reach maximum height? Explain.
   g. If the balls were launched simultaneously, would they collide before landing? Explain.

Problem 5: If a person can jump a horizontal distance of 3 m on Earth, how far could the person jump on the moon where the acceleration due to gravity is one-sixth of that on earth (1.7 m/s/s)?

Problem 6: A brick is thrown upward from the top of a building at an angle of 25 degrees above the horizontal and with an initial speed of 15 m/s. If the brick is in the air for 3 seconds, how high is the building? (Draw a picture.)

Problem 7: A daredevil tries to jump a canyon of width 10 m. To do so, he drives his motorcycle up an incline sloped at an angle of 15 degrees. What minimum speed is necessary to clear the canyon?
Problem 1 Solution:

The horizontal component of the velocity is constant since the net force in the horizontal direction is zero.
The acceleration is always downward since the net force (due to gravity) is always downward.
Since the acceleration in the vertical direction means the -y-component of the velocity is always changing, the vertical component of the velocity is initially upward, then decreases to zero at the peak, then decreases to negative (downward) velocities as it comes back down.

Problem 2 Solution:

a. The horizontal component of the velocity remains constant, 40 m/s. The vertical component of the velocity decreases by 10 m/s every second. So at the peak, the vertical component of the velocity is zero and one second later the vertical component of the velocity is -10 m/s.

b. Since the vertical component of the velocity decreases by 10 m/s every second, it will take 3 seconds for the vertical component of the velocity to slow from 30 m/s to 0 m/s, and another 3 seconds for the rock to accelerate from 0 m/s to -30 m/s. The total time in the air is 6 seconds.

c. Since the rock is in the air for six seconds and the rock moves horizontally 40 meters each second, the range for the rock is 6 s x 40 m/s = 240 m.

Problem 3 Solution:

a. Both have the same vertical acceleration since the net force is due to gravity alone (there is no horizontal acceleration.)

b. Ball A will hit farther from the building since it has a higher starting position and will therefore be in the air longer before it hits the ground. The longer it is in the air, the more time it has to move horizontally.

c. Both balls will reach the same height above their launchers since they are launched identically, but ball A will reach a greater maximum height since its launcher is above B’s launcher.

d. Ball A will be going faster. Both have the same x-velocity, but A will have a greater y-velocity since it has accelerated from a higher height.

e. Two ways are possible:

1. If the angle of the upper launcher is increased the ball will spend even more time in the air, but the x-velocity would also be decreased. The x-velocity could be decreased to the point that ball A travels only as far as ball B.

2. If the angle of the upper launcher is decreased, the x-component of the velocity would increase, but the ball will spend less time in the air. The time in the air could be decreased to the point that ball A travels only as far as ball B.

f. It is only possible if the upper launcher is not at or near 45 degrees, in which case any adjustment of the lower launcher will decrease ball B’s range.

If the upper launcher is sufficiently far from 45 degrees then two solutions are again possible:

1. If the angle of the lower launcher is increased the x-velocity would also be decreased but the ball will spend even more time in the air. The time in the air could be increased to the point that ball B travels as far as ball A.

2. If the angle of the lower launcher is decreased, the ball will spend less time in the air but the x-component of the velocity would increase. The x-component of the velocity could be increased to the point that ball B travels as far as ball A.

Problem 4 Solution:

a. Ball B since it needs to reach a greater height.

b. Ball A has the greater horizontal velocity. Since the change in height for A is less than that of B, it takes less time for A to reach the peak of its trajectory than B.

Therefore, in order for A to travel the same horizontal distance as B, but in less time, A needs a larger horizontal velocity than B.

c. B has the larger launch angle. This will give it a greater y-velocity and smaller x-velocity relative to ball A, which is necessary to satisfy the conditions stated in answers a. and b. above.

d. Both have the same vertical acceleration since the net force is due to gravity alone (there is no horizontal acceleration.)

Even if the balls were of different mass the acceleration would be the same.

e. Ball A. The diagram above is fairly carefully drawn. The parabolic trajectories must be symmetric about their midpoints (the peak of the path.) As a result, both would travel the same distance if the launch and landing points were the same height. However, since A starts at a higher position, it will get additional time in the air and will therefore travel farther horizontally.

f. Ball B. Since ball B has a greater change in vertical position, it necessarily has a greater initial vertical velocity. Since the acceleration in the vertical direction is the same for both balls, it will take longer for the ball with the greater initial vertical velocity to decelerate to an instantaneous vertical velocity of zero at the top of the flight.

g. No. The only place their paths intersect is at the top of the flight, and A reaches the top before B so they cannot collide.

Problem 5 Solution: The horizontal distance jumped is directly proportional to the time in the air (or above the ground since there is no air on the moon):

$$\Delta x = v_x \Delta t$$

The time above the ground is inversely proportional to the vertical acceleration:
\[ \Delta t = \Delta v_y / a \]

On the moon the gravitational acceleration is six times less than on Earth so the time above the ground will be six times more.

Increasing the time by a factor of six increases the horizontal displacement by a factor of six. So the person could jump six times farther, 18 m.

Problem 6 Solution: Draw a Picture:

![Diagram of a building and person jumping](image)

\( V_i = 15 \text{ m/s} \)
\( \Delta t = 3 \text{ s} \)

Find the initial vertical velocity:
\( \sin 25^\circ = V_y / 15 \text{ m/s} \)
\( V_y = 6.3 \text{ m/s} \)

Now use the y-component information to find the height:
\( \Delta y = \frac{1}{2} a \Delta t^2 + V_y \Delta t \)
\( a = -10 \text{ m/s}^2 \)
\( V_y = 6.3 \text{ m/s} \)
\( \Delta y = -45 \text{ m} + 15 \text{ m} \)
\( \Delta y = -26 \text{ m} \)

Therefore the height of the building is 26 m.

Problem 7 Solution: A daredevil tries to jump a canyon of width 10 m. To do so, he drives his motorcycle up an incline sloped at an angle of 15 degrees. What minimum speed is necessary to clear the canyon?

![Diagram of a daredevil jumping a canyon](image)

Where \( V \) represents the total velocity at the 15\(^\circ\) angle.

Solving for \( V \), \( V = \Delta x / (\Delta t \cos 15^\circ) \) \hspace{1em} Equation 1

Now write the constant velocity equation for the horizontal direction:
\( \Delta y = \frac{1}{2} a \Delta t^2 + V_y \Delta t \) where \( V_y = V \sin 15^\circ \)

Substituting for \( V_y \):
\( \Delta y = \frac{1}{2} a \Delta t^2 + V \sin 15^\circ \Delta t \) \hspace{1em} Equation 2

Now combine equations 1 and 2 by substituting for \( V \):
\( \Delta y = \frac{1}{2} a \Delta t^2 + \Delta x / (\Delta t \cos 15^\circ) \sin 15^\circ \Delta t \)

Simplify:
\( \Delta y = \frac{1}{2} a \Delta t^2 + \Delta x \sin 15^\circ / \cos 15^\circ \Delta t \)

Solve for \( \Delta t \):
\( \Delta t = \left[ \frac{2(\Delta y - \Delta x \sin 15^\circ / \cos 15^\circ)}{a} \right]^\frac{1}{2} \)

Substitute values:
\( \Delta t = \left[ \frac{2(0 \text{ m} - 10 \text{ m} \sin 15^\circ / \cos 15^\circ)}{10 \text{ m/s}^2} \right]^\frac{1}{2} \)
\( \Delta t = 0.73 \text{ s} \)

Now insert the time into equation 1 to find \( V \).
\( V = 10 \text{ m} / (0.73 \text{ s} \times \cos 15^\circ) \)
\( V = 14.18 \text{ m/s} \), the minimum speed needed for success.