CALCULUS BC
WORKSHEET 1 ON POLAR

Work the following on notebook paper.
Convert the following equations to polar form.
1. \( y = 4 \)
2. \( 3x - 5y + 2 = 0 \)
3. \( x^2 + y^2 = 25 \)

Convert the following equations to rectangular form.
4. \( r = 3 \sec \theta \)
5. \( r = 2 \sin \theta \)
6. \( \theta = \frac{5\pi}{6} \)

For the following, find \( \frac{dy}{dx} \) for the given value of \( \theta \).
7. \( r = 2 + 3 \sin \theta, \quad \theta = \frac{3\pi}{2} \)
8. \( r = 3(1 - \cos \theta), \quad \theta = \frac{\pi}{2} \)
9. \( r = 4 \sin \theta, \quad \theta = \frac{\pi}{3} \)
10. \( r = 2 \sin(3\theta), \quad \theta = \frac{\pi}{4} \)

11. Find the points of horizontal and vertical tangency for \( r = 1 + \sin \theta \). Give your answers in polar form, \( (r, \theta) \).

Use your calculator on problem 12.
12. Given the polar curve \( r = \theta + \sin(2\theta) \) for \( 0 \leq \theta \leq \pi \)
   (a) Sketch the graph of the curve.
   (b) Find the angle \( \theta \) that corresponds to the point(s) on the curve where \( x = -2 \).
   (c) Find the angle \( \theta \) that corresponds to the point(s) on the curve where \( y = 1 \).
   (d) For what values of \( \theta \), is \( \frac{dr}{d\theta} \) positive? What does this say about \( r \)?
      What does it say about the curve?

Make a table and sketch the graph.
12. \( 2 \cdot 2 \sin \cdot \) 13. \( r \cdot 1 \cdot \) 2 \( \cos \cdot \) 14. \( r \cdot 4 \cos \cdot 2 \cdot \) 15. \( r_2 \cdot 4 \sin \cdot 2 \cdot \)

CALCULUS BC
WORKSHEET 2 ON POLAR

For each of the following, sketch a graph, shade the region, and find the area.
Do not use your calculator.
1. one petal of \( r = 2 \cos(3\theta) \)
2. one petal of \( r = 4 \sin(2\theta) \)
3. interior of \( r = 2 + 2 \cos \theta \)
4. interior of \( r = 2 - \sin \theta \)

5. interior of \( r^2 = 4\sin(2\theta) \)

6. inner loop of \( r = 1 + 2\cos \theta \)

7. between the loops of \( r = 1 + 2\cos \theta \)

CALCULUS BC
WORKSHEET 3 ON POLAR

For each of the following, sketch a graph, shade the region, and find the area. Do not use your calculator.
1. inside \( r = 3\cos \theta \) and outside \( r = 2 - \cos \theta \)
2. common interior of \( r = 4\sin \theta \) and \( r = 2 \)
3. inside \( r = 3\sin \theta \) and outside \( r = 1 + \sin \theta \)
4. common interior of \( r = 3\cos \theta \) and \( r = 1 + \cos \theta \)
5. common interior of \( r = 4\sin(2\theta) \) and \( r = 2 \)

CALCULUS BC
WORKSHEET 4 ON POLAR

Work the following on notebook paper. Do not use your calculator except on problem 10.
1. Find the slope of the curve \( r = 2 - 3\sin \theta \) at the point \( (2, \pi) \).
2. Find the equation of the tangent line to the curve \( r = 3\sin(2\theta) \) at the point where \( \theta = \frac{\pi}{3} \).

On problems 3 – 5, set up an integral to find the area of the shaded region. Do not evaluate.
3. \( r = \theta \)
4. \( r = 1 + \sin \theta \)
5. \( r = 2\sin 4\theta \)
6. Sketch the polar region described by the following integral expression for area:
\[ \frac{1}{2} \int_{\alpha}^{\beta} \sin^2(3\theta) \, d\theta \]

7. (a) In polar coordinates, write equations for the line \( x = 1 \) and the circle of radius 2 centered at the origin.
(b) Write the integral in polar coordinates representing the area of the region to the right of \( x = 1 \) and inside the circle.
(c) Evaluate the integral.

8. (a) Sketch the bounded region inside the lemniscate \( r^2 = 4 \cos(2\theta) \) and outside the circle \( r = \sqrt{2} \).
(b) Compute the area of the region described in part (a).

9. Find the area between the two spirals \( r = \theta \) and \( r = 2\theta \) for \( 0 \leq \theta \leq 2\pi \).

Use your calculator on problem 10.

10. Given the polar curve \( r = \theta + 2 \sin \theta \) for \( 0 \leq \theta \leq 2\pi \)
(a) Sketch the graph of the curve.
(b) Find the angle \( \theta \) that corresponds to the point(s) on the curve where \( x = -1 \).
(c) Find the angle \( \theta \) that corresponds to the point(s) on the curve where \( y = 2 \).

CALCULUS BC
WORKSHEET 5 ON POLAR

Work the following on notebook paper. You may use your calculator on problems 2 – 5.

On problems 1 – 2, sketch a graph, shade the region, and find the area.
1. inside \( r = 2 \) and outside \( r = 2 - \sin \theta \)
2. inside \( r = 2 + 2 \cos(2\theta) \) and outside \( r = 2 \)

3. The figure shows the graphs of the line \( y = \frac{2}{3} x \) and the curve \( C \) given by \( y = \sqrt{1 - \frac{x^2}{4}} \). Let \( S \) be the region in the first quadrant bounded by the two graphs and the \( x \)-axis. The line and the curve intersect at point \( P \).
(a) Find the coordinates of \( P \).
(b) Set up and evaluate an integral expression with respect to \( x \) that gives the area of \( S \).
(c) Find a polar equation to represent curve \( C \).
(d) Use the polar equation found in (c) to set up and evaluate an integral expression with respect to the polar angle \( \theta \) that gives the area of \( S \).
4. A curve is drawn in the $xy$-plane and is described by the equation in polar coordinates

$$r = \theta + \cos(3\theta) \text{ for } \frac{\pi}{2} \leq \theta \leq \frac{3\pi}{2},$$

where $r$ is measured in meters and $\theta$ is measured in radians.

(a) Find the area bounded by the curve and the $y$-axis.
(b) Find the angle $\theta$ that corresponds to the point on the curve with $y$-coordinate $-1$.
(c) For what values of $\theta$, $\pi \leq \theta \leq \frac{3\pi}{2}$, is $\frac{dr}{d\theta}$ positive? What does this say about $r$? What does it say about the curve?
(d) Find the value of $\theta$ on the interval $\pi \leq \theta \leq \frac{3\pi}{2}$ that corresponds to the point on the curve with the greatest distance from the origin. What is the greatest distance? Justify your answer.

5. (From Calculus, 3rd edition, by Finney, Demana, Waits, Kennedy)
A region $R$ in the $xy$-plane is bounded below by the $x$-axis and above by the polar curve defined by $r = \frac{4}{1 + \sin \theta}$ for $0 \leq \theta \leq \pi$.

(a) Find the area of $R$ by evaluating an integral in polar coordinates.
(b) The curve resembles an arch of the parabola $8y = 16 - x^2$. Convert the polar equation to rectangular coordinates, and prove that the curves are the same.
(c) Set up and evaluate an integral in rectangular coordinates that gives the area of $R$.

Answers to Worksheet 1 on Polar

1. $r = 4 \csc \theta$
2. $r = \frac{-2}{3 \cos \theta - 5 \sin \theta}$
3. $r = 5$
4. $x = 3$
5. $x^2 + y^2 = 2y$
6. $y = -\frac{\sqrt{3}}{3}x$
7. 0
8. $-1$
9. $-\sqrt{3}$
10. $\frac{1}{2}$
11. Horiz: $\left(2, \frac{\pi}{2}\right), \left(\frac{1}{2}, \frac{7\pi}{6}\right), \left(\frac{1}{2}, \frac{11\pi}{6}\right)$
Vert.: $\left(\frac{3}{2}, \frac{\pi}{6}\right), \left(\frac{3}{2}, \frac{5\pi}{6}\right)$
12. (b) 2.786
(c) 0.661 and 2.223
(d) $0 \leq \theta < \frac{\pi}{3}$ and $\frac{2\pi}{3} < \theta \leq \pi$. $r$ is increasing. The curve is getting farther from the origin.
Answers to Worksheet 2 on Polar

1. $\frac{\pi}{3}$ 
2. $2\pi$ 
3. $6\pi$ 
4. $\frac{9\pi}{2}$ 
5. 4 
6. $\pi - \frac{3\sqrt{3}}{2}$ 
7. $\pi + 3\sqrt{3}$

Answers to Worksheet 3 on Polar

1. $3\sqrt{3}$ 
2. $\frac{8\pi}{3} - 2\sqrt{3}$ 
3. $\pi$ 
4. $\frac{5\pi}{4}$ 
5. $\frac{16\pi}{3} - 4\sqrt{3}$

Answers to Worksheet 4 on Polar

1. $\frac{2}{3}$ 
2. $y - \frac{9}{4} = \sqrt{3}\left(x - \frac{3\sqrt{3}}{4}\right)$ 
3. $\frac{1}{2}\int_{0}^{\pi} \theta^2 d\theta$ 
   (b) $\frac{1}{2}\int_{-\pi/3}^{\pi/3} (2^2 - \sec^2 \theta) d\theta$ 
   (b) 1.839, 4.295 
4. $\frac{1}{2}\int_{\pi/2}^{\pi} (1 + \sin \theta)^2 d\theta$ 
   (c) $\frac{4\pi - 3\sqrt{3}}{3}$ 
   (c) 0.921, 2.563 
5. $\frac{1}{2}\int_{0}^{\pi/4} (2\sin 4\theta)^2 d\theta$ 
   8. (a) graph 
   (b) $2\sqrt{3} - \frac{2\pi}{3}$

Answers to Worksheet 5 on Polar

1. $4 - \frac{\pi}{4}$ 
2. $8 + \pi$ 
3. (a) (1.2, 0.8) 
   (b) 0.927 
   (c) $r^2 = \frac{4}{4\sin^2 \theta + \cos^2 \theta}$ 
   (d) 0.927 
4. (a) 19.675 
   (b) 3.485 
   (c) $\frac{dr}{d\theta} > 0$ for $(\pi, 4.302)$. This means that the curve is getting farther from the origin. 
   (d) $\theta$ 
   $\frac{\pi}{2}$ 2.142 
   4.302 5.245 
   $\frac{3\pi}{2}$ 4.712 
   The greatest distance is 5.245 when $\theta = 4.302$. 
   (b) $r + r \sin \theta = 4$ 
   (b) $r + r \sin \theta = 4$