ASTROPHYSICS

PROBLEMS
a) Describe what is meant by a *nebula*.

an intergalactic cloud of gas and dust where all stars begin to form
EX: Some data for the variable star Betelgeuse are given below.
Average apparent brightness \( = 1.6 \times 10^{-7} \text{ Wm}^{-2} \)
Radius = 790 solar radii
Earth–Betelgeuse separation = 138 pc
The luminosity of the Sun is \( 3.8 \times 10^{26} \text{ W} \) and it has a surface temperature of 5800 K.

(a) Calculate the distance between the Earth and Betelgeuse in metres.

(b) Determine, in terms of the luminosity of the Sun, the luminosity of Betelgeuse.

(c) Calculate the surface temperature of Betelgeuse.

\[
\text{a) As } 1 \text{ pc} = 3.1 \times 10^{16} \text{ m}, \\
138 \text{ pc} = 138 \times 3.1 \times 10^{16} \\
= 4.3 \times 10^{18} \text{ m}
\]

\[
\text{b) } \frac{L}{4\pi d^2} \Rightarrow L = 4\pi d^2 b \\
= 4\pi [4.3 \times 10^{18}]^2 \times 1.6 \times 10^{-7} \\
= 3.7 \times 10^{31} \text{ W} \\
\text{Dividing by the luminosity of the Sun gives} \\
\frac{3.7 \times 10^{31}}{3.8 \times 10^{26}} = 9.7 \times 10^{4}. \\
\text{So Betelgeuse has a luminosity of } 9.7 \times 10^{4} L_{\text{Sun}}
\]

\[
\text{c) As } L = \sigma 4\pi R^2 T^4, \text{ by taking ratios we get} \\
\frac{L_{\text{Sun}}}{L_{\text{Betelgeuse}}} = \frac{\sigma 4\pi R_{\text{Betelgeuse}}^2 T_{\text{Betelgeuse}}^4}{\sigma 4\pi R_{\text{Sun}}^2 T_{\text{Sun}}^4} \\
\frac{T_{\text{Betelgeuse}}}{T_{\text{Sun}}} = \sqrt[4]{\frac{L_{\text{Betelgeuse}} R_{\text{Sun}}^2}{L_{\text{Sun}} R_{\text{Betelgeuse}}^2}} \\
= \sqrt[4]{\frac{9.8 \times 10^4}{790^2}} = 0.63 \\
T_{\text{Betelgeuse}} = 0.63 \times 5800 \text{ K} \\
= 3700 \text{ K}
\]
For a star, state the meaning of the following terms: (a) (i) Luminosity (ii) Apparent brightness

(i) The luminosity is the total power emitted by the star.

(ii) The apparent brightness is the incident power per unit area received at the surface of the Earth.

(b) The spectrum and temperature of a certain star are used to determine its luminosity to be approximately $6.0 \times 10^{31}$ W. The apparent brightness of the star is $1.9 \times 10^{-9}$ Wm$^{-2}$. These data can be used to determine the distance of the star from Earth. Calculate the distance of the star from Earth in parsec.

\[
b = \frac{L}{4\pi d^2} \Rightarrow d = \sqrt{\frac{L}{4\pi b}} = \sqrt{\frac{6.0 \times 10^{31}}{4\pi \times 1.9 \times 10^{-9}}} = 5.0 \times 10^{19} m
\]

\[
d = 5.0 \times 10^{19} m = \frac{5.0 \times 10^{19}}{9.46 \times 10^{15}} ly = \frac{5.0 \times 10^{19}}{3.26 \times 9.46 \times 10^{15}} pc = 1623 pc
\]

\[d \approx 1600 pc\]

(c) Distances to some stars can be measured by using the method of stellar parallax.

(i) Outline this method.

(ii) Modern techniques enable the measurement from Earth’s surface of stellar parallax angles as small as $5.0 \times 10^{-3}$ arcsecond. Calculate the maximum distance that can be measured using the method of stellar parallax.

\[d = \frac{1}{5 \times 10^{-3}} = 200 pc\]
EX:  

a) The star Wolf 359 has a parallax angle of 0.419 arcsecond.

(i) Describe how this parallax angle is measured.

(ii) Calculate the distance in light-year from Earth to Wolf 359

\[ d = \frac{1}{p} = \frac{1}{0.419} = 2.39 \text{ pc} = 7.78 \text{ ly} \]

(iii) State why the method of parallax can only be used for stars at a distance less than a few hundred parsec from Earth.

(iii) it is difficult to measure parallax of stars at great distances due to absorption and scattering of light in the atmosphere.

b) The ratio \( \frac{\text{apparent brightness of Wolf 359}}{\text{apparent brightness of the Sun}} \) is \( 3.7 \times 10^{-15} \).

Show that the ratio \( \frac{\text{luminosity of Wolf 359}}{\text{luminosity of the Sun}} \) is \( 8.9 \times 10^{-4} \).  

\[ b \propto \frac{L}{d^2} \text{ so } \frac{\text{luminosity of Wolf 359}}{\text{luminosity of Sun}} = 3.7 \times 10^{-15} \times \left( \frac{7.78}{1.58 \times 10^{-5}} \right)^2 = 8.9 \times 10^{-4} \]
EX:

The average intensity of the Sun’s radiation at the surface of the Earth is $1.37 \times 10^3 \text{ Wm}^{-2}$. Calculate (a) the luminosity and (b) the surface temperature of the Sun.

The mean separation of the Earth and the Sun = $1.50 \times 10^{11} \text{ m}$, radius of the Sun = $6.96 \times 10^8 \text{ m}$, Stefan–Boltzmann constant = $5.67 \times 10^{-8} \text{ Wm}^{-2} \text{ K}^{-4}$. (4 marks)

$$L = 4\pi \times (1.50 \times 10^{11})^2 \times 1.37 \times 10^3 = 3.9 \times 10^{26} \text{ W}$$

$$L = \sigma A T^4 \text{ so } T = \left(\frac{3.9 \times 10^{26}}{5.67 \times 10^{-8} \times 4\pi \times (6.96 \times 10^8)^2}\right)^\frac{1}{4} = 5.8 \times 10^3 \text{ K}$$
Since both are red (the same color), the spectra peak at the same wavelength. 

By Wien's law

\[ \lambda_{\text{max}} (\text{m}) = \frac{2.9 \times 10^{-3}}{T(\text{K})} \]

then they both have the same temperature.

\[ L = 4\pi R^2 \sigma T^4 \quad (\text{W}) \]

Star A is 9 times brighter and as they are the same distance away from Earth.

Star A is 9 times more luminous:

\[ \frac{L_A}{L_B} = \frac{4\pi R_A^2 \sigma T_A^4}{4\pi R_B^2 \sigma T_B^4} \Rightarrow 9 = \frac{R_A^2}{R_B^2} \Rightarrow R_A = 3R_B \]

So, Star A is three times bigger than star B.
**EX:** Suppose I observe with my telescope two stars, C and D, that form a binary star pair.

- Star C has a spectral peak at 350 nm - deep violet
- Star D has a spectral peak at 700 nm - deep red

What are the temperatures of the stars?

By Wien's law

\[
\lambda_{\text{peak}} = \frac{2.9 \times 10^{-3}}{T}
\]

\[
\lambda_{\text{peak in } m} = \frac{2.9 \times 10^{-3}}{T \text{ in } K}
\]

\[
T_C = \frac{2.9 \times 10^{-3}}{\lambda_{\text{peak}}} = \frac{2.9 \times 10^{-3}}{350 \times 10^{-9}} = 8300 \text{ K}
\]

\[
T_D = \frac{2.9 \times 10^{-3}}{\lambda_{\text{peak}}} = \frac{2.9 \times 10^{-3}}{350 \times 10^{-9}} = 4150 \text{ K}
\]

If both stars are equally bright (which means in this case they have equal luminosities since the stars are part of a pair the same distance away), what are the relative sizes of stars C and D?

\[
\frac{L_C}{L_D} = \frac{4\pi R_C^2 \sigma T_C^4}{4\pi R_D^2 \sigma T_D^4} \Rightarrow 1 = \frac{R_C^2 8300^4}{R_D^2 4150^4} = \frac{R_C^2}{R_D^2} (2)^4
\]

\[
R_D^2 = 16 R_C^2 \Rightarrow R_D = 4 R_C
\]

Star C is 4 times smaller than star D.
EX: (a) Explain the term black-body radiation.

Black-body radiation is that emitted by a theoretical perfect emitter for a given temperature. It includes all wavelengths of electromagnetic waves from zero to infinity.

The diagram is a sketch graph of the black-body radiation spectrum of a certain star.

(a) Copy the graph and label its horizontal axis.

(c) On your graph, sketch the black-body radiation spectrum for a star that has a lower surface temperature and lower apparent brightness than this star.

(d) The star Betelgeuse in the Orion constellation emits radiation approximating to that emitted by a black-body radiator with a maximum intensity at a wavelength of 0.97 µm. Calculate the surface temperature of Betelgeuse.

\[
T = \frac{2.9 \times 10^{-3} \text{ mK}}{\frac{\lambda_{\text{max}}}{0.97 \times 10^{-6}}} = 2.99 \times 10^3 \text{ K} \approx 3000 \text{K}
\]
Ex: (a) Define (i) luminosity (ii) apparent brightness.

(i) Luminosity is the total power radiated by star.

(ii) Apparent brightness is the power from a star received by an observer on the Earth per unit area of the observer’s instrument of observation.

(b) State the mechanism for the variation in the luminosity of the Cepheid variable.

Outer layers of the star expand and contract periodically due to interactions of the elements in a layer with the radiation emitted.

The variation with time $t$, of the apparent brightness $b$, of a Cepheid variable is shown below.

Two points in the cycle of the star have been marked A and B.

(c) (i) Assuming that the surface temperature of the star stays constant, deduce whether the star has a larger radius after two days or after six days.

(i) The radius is larger after two days (point A) because, at this time the luminosity is higher and so the star’s surface area is larger.
(ii) Explain the importance of Cepheid variables for estimating distances to galaxies.

Cepheid variables show a regular relationship between period of variation of the luminosity and the luminosity. By measuring the period the luminosity can be calculated and, by using the equation \( b = \frac{L}{4\pi d^2} \), the distances to the galaxy can be measured. This assumes that the galaxy contains the Cepheid star.

(d) (i) The maximum luminosity of this Cepheid variable is \( 7.2 \times 10^{29} \) W. Use data from the graph to determine the distance of the Cepheid variable.

\[
b = \frac{L}{4\pi d^2} \text{ thus } 1.25 \times 10^{-10} = \frac{7.2 \times 10^{29}}{4\pi d^2} \\
d = \sqrt{\frac{7.2 \times 10^{29}}{4\pi \times 1.25 \times 10^{-10}}} \quad d = 2.14 \times 10^{19} \text{ m}
\]

(ii) Cepheids are sometimes referred to as “standard candles”. Explain what is meant by this.

A standard candle is a light source of known luminosity. Measuring the period of a Cepheid allows its luminosity to be estimated. From this, other stars in the same galaxy can be compared to this known luminosity.
A partially completed Hertzsprung–Russell (HR) diagram is shown below. The line indicates the evolutionary path of the Sun from its present position, S, to its final position, F. An intermediate stage in the Sun’s evolution is labelled by I.

(a) State the condition for the Sun to move from position S.

Most of the Sun’s hydrogen has fused into helium.

(b) State and explain the change in the luminosity of the Sun that occurs between positions S and I.

Both the luminosity and the surface area increase as the Sun moves from S to I.

(c) Explain, by reference to the Chandrasekhar limit, why the final stage of the evolutionary path of the Sun is at F.

White dwarfs are found in region F of the HR diagram. Main sequence stars that end up with a mass under the Chandrasekhar limit of 1.4 solar masses will become white dwarfs.

(d) On the diagram, draw the evolutionary path of a main sequence star that has a mass of 30 solar masses.

The path must start on the main sequence above the Sun. This should lead to the super red giant region above I and either stop there or curve downwards towards and below white dwarf in the region between F and S.
EX:

a) Define *luminosity*.

total power radiated by a star

b) The sketch-graph below shows the intensity spectrum for a black body at a temperature of 6000 K.

![Intensity spectrum graph](image)

On a copy of the axes, draw a sketch-graph showing the intensity spectrum for a black body at 8000 K.

The black line intensity should be consistently higher and the maximum shown shifted to a longer wavelength.
c) A sketch of a Hertzsprung–Russell diagram is shown below.

(i) main sequence (label this M)
(ii) red giant region (label this R)
(iii) white dwarf region (label this W).

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d) In a Hertzsprung–Russell diagram, luminosity is plotted against temperature. Explain why the diagram alone does not enable the luminosity of a particular star to be determined from its temperature. (8 marks)

luminosity also depends on size of star
The diagram below shows the grid of a Hertzsprung–Russell (HR) diagram on which the positions of the Sun and four other stars A, B, C and D are shown.

a) Name the type of stars shown by A, B, C, and D.

A & D are main sequence stars; B is red giant; W is white dwarf.

b) Explain, using information from the HR diagram and without making any calculations, how astronomers can deduce that star B is larger than star A.

A and B have similar luminosities but A is far hotter. Luminosity is proportional to the area of the star and the temperature so if A is hotter than B must be larger.
c) Using the following data and information from the HR diagram, show that star B is at a distance of about 700 pc from Earth.

Apparent brightness of the Sun = \(1.4 \times 10^3\) W m\(^{-2}\)

Apparent brightness of star B = \(7.0 \times 10^{-8}\) W m\(^{-2}\)

Mean distance of the Sun from Earth = 1.0 AU

1 parsec = \(2.1 \times 10^5\) AU \hspace{1cm} (11 marks)

\[
luminosity\ of\ sun = 4\pi d^2b
\]

from HR diagram, luminosity of B = \(10^6 \times L_{sun} = 3.6 \times 10^{33}\) W

\[
d_B = \sqrt{\frac{L_B}{4\pi b}} = 6.4 \times 10^{19} \text{ m} = 700\ \text{pc}
\]
Massive stars need higher core temperatures and pressures to prevent gravitational collapse, and so fusion reactions occur at a greater rate than smaller stars.

**EX:** Briefly explain the roles of electron degeneracy, neutron degeneracy and the Chandrasekhar limit in the evolution of a star that goes supernova. (6 marks)

After a star goes supernova, left with a white dwarf: gravity is opposed by electron degeneracy pressure. However, if the star is greater than the Chandrasekhar limit, electron degeneracy pressure is not strong enough to oppose gravity, and star collapses to form a neutron star, where gravity is instead opposed by neutron degeneracy pressure.
EX:
The diagram below is a flow chart that shows the stages of evolution of a main sequence star such as the Sun. (Mass of the Sun, the solar mass = \( M_\odot \))

- main sequence star mass ≈ \( M_\odot \)
- red giant nebula
- planetary nebula
- white dwarf

a) Copy nad complete the boxes below to show the stages of evolution of a main sequence star that has a mass greater than \( 8M_\odot \).

- main sequence star mass > \( 8M_\odot \)
- red super-giant → supernova → black hole

b) Outline why:
   (i) white dwarf stars cannot have a greater mass than \( 1.4M_\odot \)
   (ii) above this mass, gravity is too strong to be opposed by electron degeneracy pressure

(ii) it is possible for a main sequence star with a mass equal to \( 8M_\odot \) to evolve into a white dwarf. (6 marks)

(ii) star can throw off mass to form planetary nebula
(a) Explain why a star having a mass of 50 times the solar mass would be expected to have a lifetime of many times less than that of the Sun.

(a) The more massive stars will have much more nuclear material (initially hydrogen). Massive stars have greater gravity so equilibrium is reached at a higher temperature at which the outward pressure due to radiation and the hot gas will balance the inward gravitational pressure. This means that fusion proceeds at a faster rate than in stars with lower mass – meaning that the nuclear fuel becomes used up far more rapidly.

(b) By referring to the mass–luminosity relationship, suggest why more massive stars will have shorter lifetimes.

(b) As the luminosity of the star is the energy used per second, stars with greater luminosity are at higher temperatures and will use up their fuel in shorter periods of time. The luminosity of a star is related to its mass by the relationship $L \propto M^{3.5}$. Therefore, increasing the mass raises the luminosity by a much larger factor which in turn means the temperature is much higher. At the higher temperature the fuel will be used in a much shorter time.
EX:
One of the most intense radio sources is the Galaxy NGC5128. Long exposure photographs show it to be a giant elliptical galaxy crossed by a band of dark dust. It lies about $1.5 \times 10^7$ light years away from Earth.

a) Describe any differences between this galaxy and the Milky Way.

Hubble’s law predicts that NGC5128 is moving away from Earth.

Milky Way is a spiral galaxy and contains more gas and dust than elliptical galaxies

b) (i) State Hubble’s law.

(i) $v = H_0 d$

(ii) State and explain what experimental measurements need to be taken in order to determine the Hubble constant.

(ii) To measure Hubble constant, plot graph of $v$ against $d$. $v$ can be obtained by measuring redshift of galaxies; $d$ can be determined using Cepheid variables

c) A possible value for the Hubble constant is $68$ km s$^{-1}$ Mpc$^{-1}$. Use this value to estimate:

(i) the recession speed of NGC5128

v = $68000 \times 4.6 = 313$ km s$^{-1}$

(ii) the age of the universe. (10 marks)

(ii) age of universe $= H_0^{-1} = 4.5 \times 10^{17}$ s
a) In an observation of a distant galaxy, spectral lines are recorded. Spectral lines at these wavelengths cannot be produced in the laboratory. Explain this phenomenon.

lines from distant galaxy are red-shifted as galaxy is receding away from us

b) Describe how Hubble's law is used to determine the distance from the Earth to distant galaxies.

Hubble's law: \( v = H_0 d \); can estimate \( v \) by determining redshift and so estimate distance \( d \).

c) Explain why Hubble’s law is not used to measure distances to nearby stars or nearby galaxies (such as Andromeda). (6 marks)

more accurate techniques exist, such as using Cepheid variables
**EX:** This question is about the Hubble constant.

(a) The value of the Hubble constant \( H_0 \) is accepted by some astronomers to be in the range 60 km s\(^{-1}\) Mpc\(^{-1}\) to 90 km s\(^{-1}\) Mpc\(^{-1}\).

(i) State and explain why it is difficult to determine a precise value of \( H_0 \)  

(ii) The Hubble constant is the constant of proportionality between the recessional velocity of galaxies and their distance from Earth. The further galaxies are away (from Earth) the more difficult it is to accurately determine how far away they are. This is because of the difficulty of both locating a standard candle, such as finding a Cepheid variable within the galaxy, and the difficulties of accurately measuring its luminosity.

(ii) State one reason why it would be desirable to have a precise value of \( H_0 \).

(1) Having a precise value of \( H_0 \) would allow us to gain an accurate value of the rate of expansion of the universe and to determine an accurate value to distant galaxies. It would also allow us to determine a more reliable value for the age of the universe.

b) The line spectrum of the light from the quasar 3C 273 contains a spectral line of wavelength 750 nm. The wavelength of the same line, measured in the laboratory, is 660 nm. Using a value of \( H_0 \) equal to 70 km s\(^{-1}\) Mpc\(^{-1}\), estimate the distance of the quasar from Earth.

\[
\Delta \lambda = 90 \times 10^{-9} \text{m}
\]

\[
z = \frac{\Delta \lambda}{\lambda_0} \approx \frac{v}{c} \rightarrow v = 4.1 \times 10^7 \text{ms}^{-1}
\]

\[
v = H_0d
\]

\[
d = \frac{4.1 \times 10^4 \text{ms}^{-1}}{70} = 590 \text{ Mpc}
\]
EX: A distant quasar is detected to have a redshift of value $z = 5.6$.

(a) Calculate the speed at which the quasar is currently moving relative to the Earth.

\[
z = \frac{\Delta \lambda}{\lambda_0} \approx \frac{v}{c} = 5.6
\]

$v = 5.6c$

(b) Estimate the ratio of the current size of the universe to its size when the quasar the emitted photons that were detected.

\[
\frac{R}{R_0} - 1 = 5.6 \implies \frac{R}{R_0} = 6.6
\]

\[
\frac{R_0}{R} = \frac{1}{6.6} = 0.15 \text{ so the universe was approximately } 15\% \text{ of its current size.}\
\]
a) State what is meant by cosmic microwave background radiation.

thermal radiation left over from the Big Bang – universe shows spectrum of blackbody emitter at around 3 K

b) Describe how the cosmic microwave background radiation provides evidence for the expanding universe. (5 marks)

CMB is isotropic (looks the same in all directions), provides evidence of the high temperature early universe that cooled as it expanded
State one piece of evidence that indicates that the Universe is expanding.

- light from distant galaxies/stars is red-shifted (which means they move away from us – as the red-shifting occurs in all directions, the universe must be expanding)
- existence of CMB
- the helium abundance in the universe which is about 25% and is consistent with a hot beginning of the universe;
EX: a) Describe the observational evidence in support of an expanding universe.
   redshift of distant galaxies; CMB

EX: No a
b) (i) Outline what is meant by *dark matter*.

  (i) evidence points to additional mass in universe; suggested that there is a dark matter *halo* surrounding the luminous matter in the universe which gives it extra mass

  (ii) Give two possible examples of dark matter.

  (ii) MACHOS: high density stars, hidden as they are far away from any luminous object
  WIMPs: non-baryonic subatomic particles, weakly interacting with baryonic matter; need huge quantities to make up dark matter