5-2 Midsegments of Triangle

Do Now
Lesson Presentation
Exit Ticket
Warm Up #6

1. Use Representations to Communicate Mathematical Ideas (1)E

Find the distance between two cities: Augusta and Brookline.

\[ d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \]

8.2

2. Find the midpoint of \( RS \) is where the endpoints are \( R(-8, 9) \) and \( S(0, -7) \). What are the coordinates of \( R \)?

\[ N(-4, 1) \]
Today we are going to learn about another part of triangles that also has a connection to similarity.

We are learning about something called a **midsegment**.

Midsegments can be found in many support structures that triangles are used in applications in the world.
5-2 Midsegments of Triangle

- A frame house
- Ladder
- Bridge
- Porch swing
- Ferris wheel
- Architecture

All supported additionally by the midsegment bars.
SWBAT

- Solve for side lengths and angle measures in similar triangles with midsegments.

**Why it matters in LIFE:** Midsegments support many structures in our world like the floors of a house, ladder, or bridge.

**Why it matters in THIS CLASS:** Sides and angles are the two fundamentals that make up triangles. Midsegments allow us to solve for both missing sides and angles in triangles.
1. Label your triangle. Label its largest angle $C$, and the other angles $A$ and $B$.

2. Fold $A$ onto $C$ to find the midpoint of $AC$.

3. Do the same for $BC$. 
4. Label the midpoints \( L \) and \( N \) and draw segment \( LN \)

5. Fold the triangle on \( LN \) and label \( D \) as shown.

6. Fold \( A \) to \( D \) and Fold \( B \) to \( D \). Label the vertices \( M \) and \( P \) as shown.
• What is the relationship between $MP$ and $AB$? How do you know?

• What conjecture can you make about the relationship between $LN$ and $AB$?
A **midsegment of a triangle** is a segment that joins the midpoints of two sides of the triangle. Every triangle has three midsegments, which form the **midsegment triangle**.

Midsegments: \( XY, YZ, ZX \)

Midsegment triangle: \( \triangle XYZ \)
Theorem 5-1  Triangle Midsegment Theorem

**Theorem**
If a segment joins the midpoints of two sides of a triangle, then the segment is parallel to the third side and is half as long.

**If . . .**
- $D$ is the midpoint of $CA$ and $E$ is the midpoint of $CB$

**Then . . .**
- $DE \parallel AB$ and $DE = \frac{1}{2}AB$

*For a proof of Theorem 5-1, see Lesson 7-3.*
A. What do you notice about the location of the library?
   - Half the distance of $SH$
   - $L(-5, 0)$ is the midpoint of $SH$

B. What do you notice about the location of Shell Gas Station?
   - $F(-2, -4)$ is the midpoint of $GH$
   - Half the distance of $GH$
**Prior Knowledge: Solve It !!!**

Suppose that your parent is picking you up from the school and drop you off at the library, then going to fill up the gas at Shell Gas Station. How far is the library to Shell Gas Station?

\[
d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
\]

\[
d(\overline{LF}) = \sqrt{(-2 - (-5))^2 + (-4 - 0)^2}
\]

\[
= \sqrt{(3)^2 + (-4)^2}
\]

\[
= \sqrt{9 + 16}
\]

\[
= \sqrt{25} \Rightarrow 5 \text{ miles}
\]
Prior Knowledge: What if? distance = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Your parent decided to pick up some groceries before returning to the school to pick up your older sibling, that is participating in after school math tutoring program, at $S(-10, 0)$. How far is the grocery store to the school?

$$d(SG) = \sqrt{(-4 - (-10))^2 + (-8 - 0)^2}$$

$$= \sqrt{(6)^2 + (-8)^2}$$

$$= \sqrt{36 + 64}$$

$$= \sqrt{100} \Rightarrow 10 \text{ miles}$$
1. What is the conjecture for point \( L \)?

\( L \) is the midpoint of \( SH \)

2. What is the conjecture for point \( F \)?

\( F \) is the midpoint of \( GH \)

3. Compare segments \( LF \) and \( SG \).

\[ LF = \frac{1}{2} SG \]

4. Justify if \( LF \parallel SG \).

Slope: \( m = \frac{y_2 - y_1}{x_2 - x_1} \)

\[ m(LF) = \frac{-4 - 0}{-2 - (-5)} = \frac{-4}{3} \]

\[ m(SG) = \frac{-8 - 0}{-4 - (-10)} = \frac{-8}{6} = \frac{-4}{3} \]
Example 1: Finding the lengths

Points $E$, $D$, and $H$ are the midpoints of the sides of $\triangle TUV$. $UV = 80$, $TV = 100$, $HD = 80$.

A. Find $HE$  
   40

B. Find $ED$  
   50

C. Find $TU$  
   160

D. Find $TE$  
   80
The triangular face of the Rock and Roll Hall of Fame in Cleveland, Ohio, is isosceles. The length of the base is 229 ft. 6 in. Each leg is divided into four congruent parts by the red segments. What is the length of the yellow segment? Explain your reasoning.
Example 2: Explain Mathematical Ideas (1)(G)

It is given that the red segments divide the legs into four congruent parts, the yellow segment is a midsegment of the triangular face of the building, so its length is one half the length of the base; therefore, the white segment is 114 ft. and 9 in.
Example 3: Use representations to Communicate Mathematical Ideas (1)(E)

You want to paddle your kayak across a lake. To determine how far you must paddle, you pace out a triangle, counting the number of strides, as shown. If your strides average 3.5 ft., what distance must you paddle across the lake?

**Plan:** multiply the strides average by the midsegment.

\[
\text{distance across the lake} = 3.5x
\]

\[
x = \frac{1}{2} (250) = 125
\]

\[
d = 3.5(125) = 437.5
\]

*the distance across the lake is 437.5 feet*
Example 4: Finding the value of $x$.

\[
8 = \frac{1}{2} (5x - 4) \quad \Delta \text{Midsegment Theorem.}
\]

\[
8 = \frac{5}{2} x - 2 \quad \text{Distributive Property}
\]

\[
10 = \frac{5}{2} x \quad \text{Combined Like Terms}
\]

\[
20 = 5x \quad \text{Multiply both sides by 2}
\]

\[
4 = x \quad \text{Divide both sides by 5}
\]
Example 5:
If $EC = 3x - 2$ and $AD = 2x + 8$. Find $EC$.

$EC = \frac{1}{2} AD \quad \Delta$ Midsegment Theorem

$3x - 2 = \frac{1}{2}(2x + 8) \quad$ Substitution

$3x - 2 = x + 4 \quad$ Distributive Property

$2x = 6 \quad$ Combined Liked Terms

$x = 3 \quad$ Divide both sides by 2.

$EC = 3(3) - 2 \quad \Leftrightarrow 7$
Got It? Solve With Your Partner

Problem 1 Finding Lengths

In the figure at the below, $AD = 6$ and $DE = 7.5$. What are the lengths of $DC$, $AC$, $EF$, and $AB$?

- $DC = 6$
- $AC = 12$
- $EF = 6$
- $AB = 15$
Verify the Triangle Midsegment Theorem for \( \triangle FGH \) with vertices \( F(-6, 4) \), \( G(4,8) \), and \( H(2, -2) \). Given that \( J \) and \( K \) are the midpoints of \( FG \) and \( FH \), respectively, show that \( JK \parallel GH \)

The midpoint of \( FG \) is \( J \left( \frac{-6 + 4}{2}, \frac{4 + 8}{2} \right) = J(-1, 6) \)

The midpoint of \( FH \) is \( K \left( \frac{-6 + 2}{2}, \frac{4 + (-2)}{2} \right) = K(-2, 1) \)

The slope of \( JK \) is \( \frac{1 - 6}{-2 - (-1)} = \frac{-5}{-1} = 5 \)

The slope of \( GH \) is \( \frac{-2 - 8}{2 - 4} = \frac{-10}{-2} = 5 \)

So \( JK \parallel GH \)

\[ JK = \sqrt{(-2 - (-1))^2 + (1 - 6)^2} = \sqrt{1 + 25} = \sqrt{26} \]

So \( JK = \frac{1}{2} GH \) with \( GH = \sqrt{(2 - 4)^2 + (-2 - 8)^2} = \sqrt{4 + 100} = \sqrt{104} = 2\sqrt{26} \)
Lesson Check  How did you Do?

1. 4

2. Each of the four small triangles has a side congruent to $\overline{JM}$, a side congruent to $\overline{NM}$, and a side congruent to $\overline{JN}$. Therefore, they are congruent by SSS.

3. The surveyor needs to measure $\overline{NM}$ to find $\overline{PQ}$. $\overline{PQ}$ is twice $\overline{NM}$.

4. A midsegment is a segment whose endpoints are the midpoints of two sides of a triangle.

5. The segments are parallel.

6. No; Susan is assuming that $L$ is the midpoint of $\overline{OT}$, which is not given.
• In the coordinate plane, how are the slopes of a midsegment and the third side of the triangle related? Explain.
  The slopes are the same because the two segments are parallel.

• How is the length of a midsegment of a triangle related to the length of the third side?
  The length of the midsegment is half the length of the third side.

• Describe the properties of midsegments of a triangle.
  • The endpoints of the segments are at the midpoints of the sides of the triangle.
  • The slopes of the midsegment and the third side of the triangle must be the same.
Exit Ticket: Apply Mathematics (1)(A)

You design a kite to look like the one at the right. Its diagonals measures 64cm and 90cm. You plan to use ribbon, represent in purple rectangle, to connect the midpoints of its sides. How much ribbon do you need?

A. 77 cm  
B. 122 cm  
C. 154 cm  
D. 308 cm