The diagram shows quadrilateral ABCD with vertices A(1, 0), B(1, 5), C(5, 2) and D(4, −1).

1a. (i) Show that \( \overrightarrow{AC} = \left( \frac{4}{2} \right) \).
(ii) Find \( \overrightarrow{BD} \).
(iii) Show that \( \overrightarrow{AC} \) is perpendicular to \( \overrightarrow{BD} \).

**Markscheme**

(i) correct approach \( A1 \)
e.g. \( \overrightarrow{OC} - \overrightarrow{OA} . \left( \frac{5}{2} \right) - \left( \frac{1}{0} \right) \)
\( \overrightarrow{AC} = \left( \frac{4}{2} \right) \) \( AG \quad N0 \)

(ii) appropriate approach \( M1 \)
e.g. \( D - B . \left( \frac{4}{-1} \right) - \left( \frac{1}{5} \right) \), move 3 to the right and 6 down
\( \overrightarrow{BD} = \left( \frac{3}{-6} \right) \) \( AI \quad N2 \)

(iii) finding the scalar product \( A1 \)
e.g. \( 4(3) + 2(-6) , 12 - 12 \)
valid reasoning \( R1 \)
e.g. \( 4(3) + 2(-6) = 0 \), scalar product is zero
\( \overrightarrow{AC} \) is perpendicular to \( \overrightarrow{BD} \) \( AG \quad N0 \)

[5 marks]

**Examiners report**
The majority of candidates were successful on part (a), finding vectors between two points and using the scalar product to show two vectors to be perpendicular.
1b. The line (AC) has equation \( \mathbf{r} = \mathbf{u} + s \mathbf{v} \).

(i) Write down vector \( \mathbf{u} \) and vector \( \mathbf{v} \).

(ii) Find a vector equation for the line (BD).

Markscheme

(i) correct “position” vector for \( \mathbf{u} \); “direction” vector for \( \mathbf{v} \)  
A1 A1 N2  
e.g. \( \mathbf{u} = \begin{pmatrix} 5 \\ 2 \end{pmatrix} \), \( \mathbf{u} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \); \( \mathbf{v} = \begin{pmatrix} 4 \\ 2 \end{pmatrix} \), \( \mathbf{v} = \begin{pmatrix} -2 \\ -1 \end{pmatrix} \)  
accept in equation e.g. \( \mathbf{a} + t \begin{pmatrix} 5 \\ -2 \end{pmatrix} \)

(ii) any correct equation in the form \( \mathbf{r} = \mathbf{a} + t \mathbf{b} \), where \( \mathbf{b} = \mathbf{BD} \)  
\[ \mathbf{r} = \begin{pmatrix} 1 \\ 5 \end{pmatrix} + t \begin{pmatrix} 3 \\ -6 \end{pmatrix}, \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ -1 \end{pmatrix} + t \begin{pmatrix} -1 \\ 2 \end{pmatrix} \]  
A2 N2

Examiners report

Although a large number of candidates answered part (b) correctly, there were many who had trouble with the vector equation of a line. Most notably, there were those who confused the position vector with the direction vector, and those who wrote their equation in an incorrect form.

1c. The lines (AC) and (BD) intersect at the point \( P(3, k) \).

Show that \( k = 1 \).

\[
3 = 1 + 4s \quad 3 = 1 + 3t
\]

\[
s = \frac{1}{2}, \quad -\frac{1}{2} \quad t = \frac{2}{3}, \quad -\frac{1}{3}
\]

\[
k = 0 + \frac{1}{2}(2)
\]

\[
k = 1
\]

\[
1 + 4s = 4 + 3t \quad 2s = -1 - 6t
\]

\[
s = \frac{1}{2}, \quad -\frac{1}{2} \quad t = \frac{2}{3}, \quad -\frac{1}{3}
\]

\[
r = \begin{pmatrix} 4 \\ -1 \end{pmatrix} - \frac{1}{3} \begin{pmatrix} 3 \\ -6 \end{pmatrix}
\]

\[
k = 1
\]
Examiners report

In part (c), most candidates seemed to know what was required, though there were many who made algebraic errors when solving for the parameters. A few candidates worked backward, using \( k = 1 \), which is not allowed on a "show that" question.

The lines (AC) and (BD) intersect at the point \( P(3, k) \).

Hence find the area of triangle ACD.

Markscheme

\[ \overrightarrow{PD} = \left( \frac{1}{2} \right) \] (A1)

\[ |\overrightarrow{PD}| = \sqrt{2^2 + 1^2} = \sqrt{5} \] (A1)

\[ |\overrightarrow{AC}| = \sqrt{4^2 + 2^2} = \sqrt{20} \] (A1)

\[ \text{area} = \frac{1}{2} \times |\overrightarrow{AC}| \times |\overrightarrow{PD}| = \frac{1}{2} \times \sqrt{20} \times \sqrt{5} \] (MI)

\[ = 5 \] (A1) N4

[5 marks]

Examiners report

In part (d), candidates attempted many different geometric and vector methods to find the area of the triangle. As the question said "hence", it was required that candidates should use answers from their previous working - i.e. AC\( \perp \)BD and \( P(3,1) \). Some geometric approaches, while leading to the correct answer, did not use "hence" or lacked the required justification.

The line \( L_1 \) passes through the points \( P(2,4,8) \) and \( Q(4,5,4) \).

2.a. (i) Find \( \overrightarrow{PQ} \). [4 marks]

(ii) Hence write down a vector equation for \( L_1 \) in the form \( r = a + sb \).

Markscheme

(i) evidence of approach (MI)

\[ \text{e.g. } \overrightarrow{PO} + \overrightarrow{OQ}, \overrightarrow{P} - \overrightarrow{Q} \]

\[ \overrightarrow{PQ} = \left( \begin{array}{c} 2 \\ 1 \\ -4 \end{array} \right) \] (A1) N2

(ii) any correct equation in the form \( r = a + sb \) (accept any parameter for \( s \))

where \( a \) is \( \left( \begin{array}{c} 2 \\ 4 \\ 8 \end{array} \right) \) or \( \left( \begin{array}{c} 4 \\ 5 \\ 4 \end{array} \right) \), and \( b \) is a scalar multiple of \( \left( \begin{array}{c} 2 \\ 1 \\ -4 \end{array} \right) \) (A2) N2

\[ \text{e.g. } r = \left( \begin{array}{c} 2 \\ 4 \\ 8 \end{array} \right) + s \left( \begin{array}{c} 2 \\ 1 \\ -4 \end{array} \right), \quad r = \left( \begin{array}{c} 4 + 2s \\ 5 + 1s \\ 4 - 4s \end{array} \right) \]

\[ r = 2i + 4j + 8k + s(2i + 1j - 4k) \]

Note: Award A1 for the form \( a + sb \), A1 for \( L = a + sb \), A0 for \( r = b + sa \). [4 marks]
Examiners report

In part (a), nearly all the candidates correctly found the vector PQ, and the majority went onto find the correct vector equation of the line. There are still many candidates who do not write this equation in the correct form, using "r = ", and these candidates were penalized one mark.

Examiners report

The line $L_2$ is perpendicular to $L_1$, and parallel to $\begin{pmatrix} 3p \\ 2p \\ 4 \end{pmatrix}$, where $p \in \mathbb{Z}$.

(i) Find the value of $p$. 

(ii) Given that $L_2$ passes through $R(10, 6, -40)$, write down a vector equation for $L_2$.

Markscheme

(i) choosing correct direction vectors for $L_1$ and $L_2$ (AI) (AI)

e.g. $\begin{pmatrix} 2 \\ 1 \\ -4 \end{pmatrix}$, $\begin{pmatrix} 3p \\ 2p \\ 4 \end{pmatrix}$

evidence of equating scalar product to 0 (M1)
correct calculation of scalar product AI

e.g. $2 \times 3p + 1 \times 2p + (-4) \times 4$, $8p - 16 = 0$

$p = 2$ AI N3

(ii) any correct expression in the form $r = a + tb$ (accept any parameter for $t$)

where $a$ is $\begin{pmatrix} 10 \\ 6 \\ -40 \end{pmatrix}$, and $b$ is a scalar multiple of $\begin{pmatrix} 6 \\ 4 \end{pmatrix}$ A2 N2

e.g. $r = \begin{pmatrix} 10 \\ 6 \\ -40 \end{pmatrix} + t \begin{pmatrix} 6 \\ 4 \end{pmatrix}$, $r = \begin{pmatrix} 10 + 6s \\ 6 + 4s \\ -40 + 4s \end{pmatrix}$, $r = 10i + 6j - 40k + s(6i + 4j + 4k)$

Note: Award AI for the form $a + tb$, AI for $L = a + tb$ (unless they have been penalised for $L = a + sb$ in part (a)), A0 for $r = b + ta$.

[7 marks]

Examiners report

In part (b), the majority of candidates knew to set the scalar product equal to zero for the perpendicular vectors, and were able to find the correct value of $p$.

[7 marks]

2c. The lines $L_1$ and $L_2$ intersect at the point A. Find the $x$-coordinate of A.
**Markscheme**

appropriate approach  \((M1)\)

e.g. \[
\begin{pmatrix}
2 \\
4 \\
8
\end{pmatrix}
+ s
\begin{pmatrix}
2 \\
1 \\
-4
\end{pmatrix}
= \begin{pmatrix}
10 \\
6 \\
-40
\end{pmatrix}
+ t
\begin{pmatrix}
6 \\
4 \\
4
\end{pmatrix}
\]

any two correct equations with different parameters  \(A1\)

e.g.  
\[2 + 2s = 10 + 6t, \quad 4 + s = 6 + 4t, \quad 8 - 4s = -40 + 4t\]

attempt to solve simultaneous equations  \((M1)\)

correct working  \((A1)\)

e.g.  
\[s = -2 - 2t, \quad 4 = 2t, \quad -4 + 5s = 46, \quad 5s = 50\]

one correct parameter  \(s = 10, t = 2\)  \(A1\)

\(x = 22\) (accept \((22, 14, -32)\))  \(A1\)  \(N4\)

[7 marks]

**Examiners report**

A good number of candidates used the correct method to find the intersection of the two lines, though some algebraic and arithmetic errors kept some from finding the correct final answer.

\[
\begin{align*}
A \text{ line } L \text{ passes through } A(1, -1, 2) \text{ and is parallel to the line } & r = \begin{pmatrix}
-2 \\
1 \\
5
\end{pmatrix} + s
\begin{pmatrix}
1 \\
3 \\
-2
\end{pmatrix} .
\end{align*}
\]

Write down a vector equation for \(L\) in the form  \(r = a + tb\) .  \([2 \text{ marks}]\)

**Markscheme**

correct equation in the form  \(r = a + tb\)  \(A2\)  \(N2\)

\[
\begin{align*}
r &= \begin{pmatrix}
1 \\
-1 \\
2
\end{pmatrix} + t
\begin{pmatrix}
1 \\
3 \\
-2
\end{pmatrix} 
\end{align*}
\]

[2 marks]

**Examiners report**

Many candidates answered this question well. Some continue to write the vector equation in (a) using "\(L = \)", which does not earn full marks.

The line \(L\) passes through point \(P\) when  \(t = 2\) .

**3b.** Find

(i) \(\overrightarrow{OP}\) ;

(ii) \(|\overrightarrow{OP}|\) .  \([4 \text{ marks}]\)
**Markscheme**

(i) attempt to substitute \( t = 2 \) into the equation \((M1)\)

\[
\begin{pmatrix}
2 \\
6 \\
-4
\end{pmatrix} + \begin{pmatrix}
1 \\
-1 \\
2
\end{pmatrix} + \begin{pmatrix}
1 \\
3 \\
-2
\end{pmatrix}
\]

\[
\overrightarrow{OP} = \begin{pmatrix}
3 \\
5 \\
-2
\end{pmatrix} \quad A1 \quad N2
\]

(ii) correct substitution into formula for magnitude \( A1 \)

\[
\sqrt{3^2 + 5^2 + (-2)^2} = |\overrightarrow{OP}| = \sqrt{38} \quad A1 \quad N1
\]

[4 marks]

**Examiners report**

Part (b) proved accessible for most, although small arithmetic errors were not uncommon. Some candidates substituted \( t = 2 \) into the original equation, and a few answered \( \overrightarrow{OP} = \begin{pmatrix}
2 \\
6 \\
-4
\end{pmatrix} \). A small but surprising number of candidates left this question blank, suggesting the topic was not given adequate attention in course preparation.

---

The following diagram shows the obtuse-angled triangle ABC such that \( \overrightarrow{AB} = \begin{pmatrix}
-3 \\
0 \\
-4
\end{pmatrix} \) and \( \overrightarrow{AC} = \begin{pmatrix}
-2 \\
2 \\
-6
\end{pmatrix} \).

4a. (i) Write down \( \overrightarrow{BA} \).

(ii) Find \( \overrightarrow{BC} \).

[3 marks]
**Markscheme**

(i) \( \overrightarrow{BA} = \begin{pmatrix} 3 \\ 0 \\ 4 \end{pmatrix} \)  \( A1 \)  \( N1 \)

(ii) evidence of combining vectors  \( (M1) \)

\[
\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC} , \overrightarrow{BA} + \overrightarrow{AC} , \begin{pmatrix} -2 \\ 2 \\ -6 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ -4 \end{pmatrix}
\]

\( \overrightarrow{BC} = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix} \)  \( A1 \)  \( N2 \)

[3 marks]

**Examiners report**

Many candidates answered (a) correctly, although some reversed the vectors when finding \( \overrightarrow{BC} \), while others miscopied the vectors from the question paper.

4b.  

(i) Find \( \cos \overrightarrow{ABC} \).

(ii) Hence, find \( \sin \overrightarrow{ABC} \).

**Markscheme**

(i) **METHOD 1**

finding \( \overrightarrow{BA} \bullet \overrightarrow{BC} \), \(|\overrightarrow{BA}|\), \(|\overrightarrow{BC}|\)

\[
\overrightarrow{BA} \bullet \overrightarrow{BC} = 3 \times 1 + 0 + 4 \times -2 , \quad |\overrightarrow{BA}| = \sqrt{3^2 + 4^2} \quad , \quad |\overrightarrow{BC}| = 3
\]

substituting into formula for \( \cos \theta \)  \( M1 \)

\[
e.g. \quad \frac{2 \times 1 + 0 + 4 \times -2}{3\sqrt{3^2 + 4^2}} , \quad \frac{5}{7 \times 3}
\]

\[
\cos \overrightarrow{ABC} = -\frac{5}{15} \quad (= -\frac{1}{3}) \quad A1 \)  \( N3 \)

**METHOD 2**

finding \( |\overrightarrow{AC}| \), \(|\overrightarrow{BA}|\), \(|\overrightarrow{BC}|\)  \( (A1)(A1)(A1) \)

\[
|\overrightarrow{AC}| = \sqrt{2^2 + 2^2 + 6^2} \quad , \quad |\overrightarrow{BA}| = \sqrt{3^2 + 4^2} \quad , \quad |\overrightarrow{BC}| = 3
\]

substituting into cosine rule  \( M1 \)

\[
e.g. \quad \frac{25 + 9 - 44}{2 \times 5 \times 3} , \quad \frac{25 + 9 - 44}{90}
\]

\[
\cos \overrightarrow{ABC} = -\frac{10}{90} \quad (= -\frac{1}{9}) \quad A1 \)  \( N3 \)

(ii) evidence of using Pythagoras  \( (M1) \)

\[
e.g. \quad \text{right-angled triangle with values, } \sin^2 x + \cos^2 x = 1
\]

\[
\sin \overrightarrow{ABC} = \frac{\sqrt{8}}{3} \quad (= \frac{2\sqrt{2}}{3}) \quad A1 \)  \( N2 \)

[7 marks]

**Examiners report**

Students had no difficulty finding the scalar product and magnitudes of the vectors used in finding the cosine. However, few recognized that \( \overrightarrow{BA} \) is the vector to apply in the formula to find the cosine value. Most used \( \overrightarrow{AB} \) to obtain a positive cosine, which neglects that the angle is obtuse and thus has a negative cosine. Surprisingly few students could then take a value for cosine and use it to find a value for sine. Most left (bii) blank entirely.
4c. The point D is such that \( \overrightarrow{CD} = \left( \begin{array}{c} -4 \\ 5 \\ p \end{array} \right) \), where \( p > 0 \).

(i) Given that \( |\overrightarrow{CD}| = \sqrt{50} \), show that \( p = 3 \).

(ii) Hence, show that \( \overrightarrow{CD} \) is perpendicular to \( \overrightarrow{BC} \).

**Markscheme**

(i) attempt to find an expression for \( |\overrightarrow{CD}| \) \( (M1) \)

e.g. \( \sqrt{(-4)^2 + 5^2 + p^2} \), \( |\overrightarrow{CD}|^2 = 4^2 + 5^2 + p^2 \)
correct equation \( A1 \)
e.g. \( \sqrt{(-4)^2 + 5^2 + p^2} = \sqrt{50} \), \( 4^2 + 5^2 + p^2 = 50 \)
p\(^2 = 9 \) \( A1 \)
p = 3 \( AG \ N0 \)

(ii) evidence of scalar product \( (M1) \)

e.g. \( \left( \begin{array}{c} -4 \\ 5 \\ 3 \end{array} \right) \cdot \left( \begin{array}{c} 1 \\ 2 \\ -2 \end{array} \right) \), \( \overrightarrow{CD} \cdot \overrightarrow{BC} \)
correct substitution

e.g. \( -4 \times 1 + 5 \times 2 + 3 \times -2 \), \( -4 + 10 - 6 \) \( A1 \)
\( \overrightarrow{CD} \cdot \overrightarrow{BC} = 0 \) \( A1 \)
\( \overrightarrow{CD} \) is perpendicular to \( \overrightarrow{BC} \) \( AG \ N0 \)

[6 marks]

**Examiners report**

Part (c) proved accessible for many candidates. Some created an expression for \( |\overrightarrow{CD}| \) and then substituted the given \( p = 3 \) to obtain \( \sqrt{50} \), which does not satisfy the “show that” instruction. Many students recognized that the scalar product must be zero for vectors to be perpendicular, and most provided the supporting calculations.

The following diagram shows quadrilateral ABCD, with \( \overrightarrow{AD} = \overrightarrow{BC} \), \( \overrightarrow{AB} = \left( \begin{array}{c} 3 \\ 1 \end{array} \right) \), and \( \overrightarrow{AC} = \left( \begin{array}{c} 4 \\ 4 \end{array} \right) \).

5a. Find \( \overrightarrow{BC} \). 

[2 marks]
Markscheme

evidence of appropriate approach \((M1)\)
e.g. \(\overrightarrow{AC} - \overrightarrow{AB}, \begin{pmatrix} 4 - 3 \\ 4 - 1 \end{pmatrix}\)
\[
\overrightarrow{BC} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} \quad AI \quad N2
\]

[2 marks]

Examiners report

This question on two-dimensional vectors was generally very well done.

5b. Show that \(\overrightarrow{BD} = \begin{pmatrix} -2 \\ 2 \end{pmatrix}\). [2 marks]

Markscheme

METHOD 1
\[
\overrightarrow{AD} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} \quad (AI)
\]
correct approach \(AI\)
e.g. \(\overrightarrow{AD} - \overrightarrow{AB}, \begin{pmatrix} 1 - 3 \\ 3 - 1 \end{pmatrix}\)
\[
\overrightarrow{BD} = \begin{pmatrix} -2 \\ 2 \end{pmatrix} \quad AG \quad N0
\]

METHOD 2
recognizing \(\overrightarrow{CD} = \overrightarrow{BA}\) \((AI)\)
correct approach \(AI\)
e.g. \(\overrightarrow{BC} + \overrightarrow{CD}, \begin{pmatrix} 1 - 3 \\ 3 - 1 \end{pmatrix}\)
\[
\overrightarrow{BD} = \begin{pmatrix} -2 \\ 2 \end{pmatrix} \quad AG \quad N0
\]

[2 marks]

Examiners report

This question on two-dimensional vectors was generally very well done. A very small number of candidates had trouble with the "show that" in part (b) of the question.

5c. Show that vectors \(\overrightarrow{BD}\) and \(\overrightarrow{AC}\) are perpendicular. [3 marks]
Markscheme

METHOD 1
evidence of scalar product \((MI)\)
e.g. \(\text{BD} \cdot \text{AC} , \begin{pmatrix} -2 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ 4 \end{pmatrix}\)
correct substitution \(A1\)
e.g. \((-2)(4) + (2)(4) , -8 + 8\)
\(\text{BD} \cdot \text{AC} = 0 \quad A1\)
therefore vectors \(\text{BD}\) and \(\text{AC}\) are perpendicular \(AG\) \(N0\)

METHOD 2
try to find angle between two vectors \((MI)\)
e.g. \(\frac{\text{BD} \cdot \text{AC}}{\|\text{BD}\| \|\text{AC}\|}\)
correct substitution \(A1\)
e.g. \(\frac{(-2)(4) + (2)(4)}{\sqrt{32}} , \cos \theta = 0\)
\(\theta = 90^\circ \quad A1\)
therefore vectors \(\text{BD}\) and \(\text{AC}\) are perpendicular \(AG\) \(N0\)

[3 marks]

Examiners report

Nearly all candidates knew to use the scalar product in part (c) to show that the vectors are perpendicular.

Examiners report

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Examiners report

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6a. Find \(\overrightarrow{AB}\). \([2\text{ marks}]\)

Markscheme

appropriate approach \((MI)\)
e.g. \(\overrightarrow{AO} + \overrightarrow{OB} , B - A\)
\(\overrightarrow{AB} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \quad A1\) \(N2\)

[2 marks]

Examiners report

Finding \(\overrightarrow{AB}\) was generally well done, although some candidates reversed the subtraction.

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6b. Find an equation for \(L_1\) in the form \(r = a + tb\). \([2\text{ marks}]\)
**Markscheme**

any correct equation in the form \( r = a + tb \) \( A2 \) \( N2 \)

where \( b \) is a scalar multiple of \( \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \)

e.g. \( r = \begin{pmatrix} 1 \\ -1 \\ \frac{1}{4} \end{pmatrix} + t \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \), \( r = \begin{pmatrix} 2 + t \\ -2 - t \\ 5 + t \end{pmatrix} \), \( r = 2i - 2j + 5k + t(i - j + k) \)

[2 marks]

**Examiners report**

In part (b) not all the candidates recognized that \( \vec{AB} \) was the direction vector of the line, as some used the position vector of point \( B \) as the direction vector.

Line \( L_2 \) has equation \( r = \begin{pmatrix} 2 \\ 4 \\ 7 \end{pmatrix} + s \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} \).

Find the angle between \( L_1 \) and \( L_2 \).

**Markscheme**

choosing correct direction vectors \( \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} , \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \) \( (A1)(A1) \)

finding scalar product and magnitudes \( (A1)(A1)(A1) \)

scalar product \( = 1 \times 2 + -1 \times 1 + 1 \times 3 \) \( (= 4) \)

magnitudes \( \sqrt{1^2 + (-1)^2 + 1^2} \) \( (= 1.73 \ldots) \), \( \sqrt{4 + 1 + 9} \) \( (= 3.74 \ldots) \)

substitution into \( \cos \theta = \frac{\vec{u} \cdot \vec{v}}{|\vec{u}| \cdot |\vec{v}|} \) \( M1 \)

e.g. \( \cos \theta = \frac{1 \times 2 + -1 \times 1 + 1 \times 3}{\sqrt{2^2 + (-1)^2 + 3^2}} \), \( \cos \theta = \frac{4}{\sqrt{42}} \)

\( \theta = 0.906 \) \( (51.9^\circ) \) \( A1 \) \( N5 \)

[7 marks]

**Examiners report**

Many candidates successfully used scalar product and magnitudes in part (c), although a large number did choose vectors other than the direction vectors and many did not state clearly which vectors they were using.

The lines \( L_1 \) and \( L_2 \) intersect at point \( C \). Find the coordinates of \( C \).

[6 marks]
7a. (i) Find the velocity vector, \( \overrightarrow{AB} \).

(ii) Find the speed of the particle.
**Markscheme**

(i) evidence of approach  \((M1)\)

\[\overrightarrow{AO} + \overrightarrow{OB} , B - A , \begin{pmatrix} 9 - 6 \\ -6 + 2 \\ 15 - 10 \end{pmatrix} \]

\[\overrightarrow{AB} = \begin{pmatrix} 3 \\ -4 \\ 5 \end{pmatrix} \text{ (accept } (3, -4, 5) \text{ ) } \text{ AI N2} \]

(ii) evidence of finding the magnitude of the velocity vector  \((M1)\)

\[\text{e.g. speed } = \sqrt{3^2 + 4^2 + 5^2} \]

\[\text{speed } = \sqrt{50} \text{ ( } = 5\sqrt{2} \text{ ) } \text{ AI N1} \]

[4 marks]

**Examiners report**

This question was quite well done. Marks were lost when candidates found the vector \(\overrightarrow{BA}\) instead of \(\overrightarrow{AB}\) in part (a) and for not writing their vector equation as an equation.

---

7b. Write down an equation of the line \(L\).

[2 marks]

**Markscheme**

correct equation (accept Cartesian and parametric forms)  \((A2 \text{ N2})\)

\[\begin{align*} 
\text{e.g. } r &= \begin{pmatrix} 6 \\ -2 \\ 10 \end{pmatrix} + t \begin{pmatrix} 3 \\ -4 \\ 5 \end{pmatrix} , \\
\text{e.g. } r &= \begin{pmatrix} 9 \\ -6 \\ 15 \end{pmatrix} + t \begin{pmatrix} 3 \\ -4 \\ 5 \end{pmatrix} 
\end{align*} \]

[2 marks]

**Examiners report**

In part (b), a few candidates switched the position and velocity vectors or used the vectors \(\overrightarrow{OA}\) and \(\overrightarrow{OB}\) to incorrectly write the vector equation.
The diagram shows a parallelogram ABCD.

The coordinates of A, B and D are A(1, 2, 3), B(6, 4, 4) and D(2, 5, 5).

8a. [5 marks]

(i) Show that \[ \overrightarrow{AB} = \begin{pmatrix} 5 \\ 2 \\ 1 \end{pmatrix} \].

(ii) Find \( \overrightarrow{AD} \).

(iii) Hence show that \( \overrightarrow{AC} = \begin{pmatrix} 6 \\ 5 \\ 3 \end{pmatrix} \).

---

**Markscheme**

(i) evidence of approach \( M1 \)

\[ B - A , \overrightarrow{AO} + \overrightarrow{OB} , \begin{pmatrix} 6 \\ 4 \\ 4 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \]

\[ \overrightarrow{AB} = \begin{pmatrix} 5 \\ 2 \\ 1 \end{pmatrix} AG N0 \]

(ii) evidence of approach \( M1 \)

\[ D - A , \overrightarrow{AO} + \overrightarrow{OD} , \begin{pmatrix} 2 \\ 5 \\ 5 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \]

\[ \overrightarrow{AD} = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} AI N2 \]

(iii) evidence of approach \( M1 \)

\[ \overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{AD} \]

correct substitution \( AI \)

\[ \overrightarrow{AC} = \begin{pmatrix} 5 \\ 2 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} \]

\[ \overrightarrow{AC} = \begin{pmatrix} 6 \\ 5 \\ 3 \end{pmatrix} AG N0 \]

[5 marks]

**Examiners report**

Candidates performed very well in this question, showing a strong ability to work with the algebra and geometry of vectors.

---

8b. Find the coordinates of point C. [3 marks]
Markscheme

evidence of combining vectors (there are at least 5 ways) \((M1)\)
e.g. \(\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{AC} \)
\(\overrightarrow{OC} = \overrightarrow{OB} + \overrightarrow{AD} \)
\(\overrightarrow{AB} = \overrightarrow{OC} - \overrightarrow{OD} \)
correct substitution \(A1\)
\[
\overrightarrow{OC} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \begin{pmatrix} 6 \\ 5 \\ 3 \end{pmatrix} = \begin{pmatrix} 7 \\ 7 \\ 6 \end{pmatrix}
\]
e.g. coordinates of C are \((7, 7, 6)\) \(AI \ N1\)

[3 marks]

Examiners report

Candidates performed very well in this question, showing a strong ability to work with the algebra and geometry of vectors.

8c. (i) Find \(\overrightarrow{AB} \cdot \overrightarrow{AD} \).

(ii) \textbf{Hence} find angle \(A\).

Markscheme

(i) evidence of using scalar product on \(\overrightarrow{AB} \) and \(\overrightarrow{AD} \) \((M1)\)
e.g. \(\overrightarrow{AB} \cdot \overrightarrow{AD} = 5(1) + 2(3) + 1(2) \)
\(\overrightarrow{AB} \cdot \overrightarrow{AD} = 13 \) \(A1 \ N2\)
(ii) \(\|\overrightarrow{AB}\| = 5.477\ldots \) \(\|\overrightarrow{AD}\| = 3.741\ldots \) \((A1)(A1)\)
evidence of using \(\cos A = \frac{\overrightarrow{AB} \cdot \overrightarrow{AD}}{\|\overrightarrow{AB}\| \|\overrightarrow{AD}\|} \) \((M1)\)
correct substitution \(A1\)
e.g. \(\cos A = \frac{13}{20.493} \)
\(\hat{A} = 0.884\) \((50.6^\circ)\) \(AI \ N3\)

[7 marks]

Examiners report

Some candidates were unable to find the scalar product in part (c), yet still managed to find the correct angle, able to use the formula in the information booklet without knowing that the scalar product is a part of that formula.

8d. Hence, or otherwise, find the area of the parallelogram. \([3 \text{ marks}]\)
**Markscheme**

**METHOD 1**
evidence of using \( \text{area} = 2 \left( \frac{1}{2} |\overrightarrow{AD}| |\overrightarrow{AB}| \sin \theta \right) \) \((M1)\)
correct substitution \( A1 \)
e.g. area = 2 \( \left( \frac{1}{2} (3, 741 \ldots) (5.477 \ldots) \sin 0.883 \ldots \right) \) \( A1 \) \( N2 \)

**METHOD 2**
evidence of using \( b \times h \) \((M1)\)
finding height of parallelogram \( A1 \)
e.g. \( h = 3.741 \ldots \times \sin 0.883 \ldots (= 2.892 \ldots) \), \( h = 5.477 \ldots \times \sin 0.883 \ldots (= 4.234 \ldots) \) \( A1 \) \( N2 \)

[3 marks]

**Examiners report**

Few candidates considered that the area of the parallelogram is twice the area of a triangle, which is conveniently found using \( \overrightarrow{B\hat{A}\hat{D}} \). In an effort to find base \( \times \) height, many candidates multiplied the magnitudes of \( \overrightarrow{AB} \) and \( \overrightarrow{AD} \), missing that the height of a parallelogram is perpendicular to a base.

The line \( L_1 \) is represented by the vector equation \( r = \begin{pmatrix} -3 \\ -1 \\ -25 \end{pmatrix} + p \begin{pmatrix} 2 \\ 1 \\ -8 \end{pmatrix} \).

A second line \( L_2 \) is parallel to \( L_1 \) and passes through the point \( B(-8, -5, 25) \).

9a. Write down a vector equation for \( L_2 \) in the form \( r = a + tb \). \([2 \text{ marks}]\)

**Markscheme**

any correct equation in the form \( r = a + tb \) (accept any parameter) \( A2 \) \( N2 \)
e.g. \( r = \begin{pmatrix} -8 \\ -5 \\ 25 \end{pmatrix} + t \begin{pmatrix} 2 \\ 1 \\ -8 \end{pmatrix} \)

Note: Award \( A1 \) for \( a + tb \), \( \text{A1} \) for \( L = a + tb \), \( A0 \) for \( r = b + ta \).

[2 marks]

**Examiners report**

Many candidates gave a correct vector equation for the line.

9b. A third line \( L_3 \) is perpendicular to \( L_1 \) and is represented by \( r = \begin{pmatrix} 5 \\ 0 \\ 3 \end{pmatrix} + q \begin{pmatrix} -7 \\ -2 \\ k \end{pmatrix} \).

Show that \( k = -2 \). \([5 \text{ marks}]\)
**Markscheme**
recognizing scalar product must be zero (seen anywhere)  \( R1 \)
e.g. \( \mathbf{a} \cdot \mathbf{b} = 0 \)
evidence of choosing direction vectors \( \begin{pmatrix} 2 \\ 1 \\ -8 \end{pmatrix}, \begin{pmatrix} -7 \\ -2 \\ k \end{pmatrix} \) (\( AI \))(\( AI \))
correct calculation of scalar product  \( (AI) \)
e.g. \( 2(-7) + 1(-2) - 8k \)
simplification that clearly leads to solution  \( AI \)
e.g. \(-16 - 8k \), \(-16 - 8k = 0 \)
\( k = -2 \)  \( AG \)  \( N0 \)

[5 marks]

**Examiners report**
A common error was to misplace the initial position and direction vectors. Those who set the scalar product of the direction vectors to zero typically solved for \( k \) successfully. Those who substituted \( k = -2 \) earned fewer marks for working backwards in a “show that” question.

9c.

The lines \( L_1 \) and \( L_3 \) intersect at the point A.

Find the coordinates of A.

**Markscheme**
evidence of equating vectors  \( (MI) \)
e.g. \( L_1 = L_3 \), \( \begin{pmatrix} -3 \\ -1 \\ -25 \end{pmatrix} + p \begin{pmatrix} 2 \\ 1 \\ -8 \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \\ 3 \end{pmatrix} + q \begin{pmatrix} -7 \\ -2 \\ -2 \end{pmatrix} \)
any two correct equations  \( AI \)
e.g. \(-3 + 2p = 5 - 7q \), \(-1 + p = -2q \), \(-25 - 8p = 3 - 2q \)
attempting to solve equations  \( (MI) \)
finding one correct parameter \( (p = -3 , q = 2) \)  \( AI \)
the coordinates of A are \((-9,-4,-1) \)  \( AI \)  \( N3 \)

[6 marks]

**Examiners report**
Many went on to find the coordinates of point A, however some used the same letter, say \( p \), for each parameter and thus could not solve the system.

9d.

The lines \( L_2 \) and \( L_3 \) intersect at point C where \( \mathbf{BC} = \begin{pmatrix} 6 \\ 3 \\ -24 \end{pmatrix} \).

(i) Find \( \mathbf{AB} \).
(ii) Hence, find \( |\mathbf{AC}| \).
**Markscheme**

(i) evidence of appropriate approach \((M1)\)

\[ \overrightarrow{OA} + \overrightarrow{AB} = \overrightarrow{OB} \]

\[ \overrightarrow{AB} = \begin{pmatrix} \frac{1}{26} \\ -1 \\ \frac{1}{26} \end{pmatrix} \]

\( \text{A1} \ N2 \)

(ii) finding \( \overrightarrow{AC} \)

\( \overrightarrow{AC} = \begin{pmatrix} 7 \\ 2 \\ 2 \end{pmatrix} \)

\( \text{A1} \)

evidence of finding magnitude \((M1)\)

\[ |\overrightarrow{AC}| = \sqrt{7^2 + 2^2 + 2^2} \]

\[ |\overrightarrow{AC}| = \sqrt{57} \]

\( \text{A1} \ N3 \)

[5 marks]

**Examiners report**

Part (d) proved challenging as many candidates did not consider that \( \overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC} \). Rather, many attempted to find the coordinates of point C, which became a more arduous and error-prone task.

Let \( \overrightarrow{AB} = \begin{pmatrix} 6 \\ -2 \\ 3 \end{pmatrix} \) and \( \overrightarrow{AC} = \begin{pmatrix} -2 \\ -3 \\ 2 \end{pmatrix} \).

\[ [2 \text{ marks}] \]

**Markscheme**

10a. Find \( \overrightarrow{BC} \).

\( \text{A1} \ N2 \)

[2 marks]

**Examiners report**

Part (a) was generally done well with candidates employing different correct methods to find the vector \( \overrightarrow{BC} \). Some candidates subtracted the given vectors in the wrong order and others simply added them. Calculation errors were seen with some frequency.

10b. Find a unit vector in the direction of \( \overrightarrow{AB} \).

\[ [3 \text{ marks}] \]
Markscheme

attempt to find the length of $\vec{AB}$  \((M1)\)

$|\vec{AB}| = \sqrt{6^2 + (-2)^2 + 3^2} = \sqrt{36 + 4 + 9} = \sqrt{49} = 7$  \((A1)\)

unit vector is \(\frac{1}{7} \begin{pmatrix} 6 \\ -2 \\ 3 \end{pmatrix} = \begin{pmatrix} \frac{6}{7} \\ -\frac{2}{7} \\ \frac{3}{7} \end{pmatrix} \) \(A1\) \(N2\)

[3 marks]

Examiners report

Many candidates did not appear to know how to find a unit vector in part (b). Some tried to write down the vector equation of a line, indicating no familiarity with the concept of unit vectors while others gave the vector \((1, 1, 1)\) or wrote the same vector as a linear combination of \(i, j\) and \(k\). A number of candidates correctly found the magnitude but did not continue on to write the unit vector.

10c. Show that $\vec{AB}$ is perpendicular to $\vec{AC}$.

[3 marks]

Markscheme

recognizing that the dot product or $\cos \theta$ being 0 implies perpendicular  \((M1)\)

correct substitution in a scalar product formula  \(A1\)

e.g. \((6) \times (-2) + (-2) \times (-3) + (3) \times (2)\)  \(\cos \theta = \frac{-12 + 6 + 6}{7 \sqrt{17}}\)

correct calculation  \(A1\)

e.g. $\vec{AB} \cdot \vec{AC} = 0$  \(\cos \theta = 0\)

therefore, they are perpendicular  \(AG\) \(N0\)

[3 marks]

Examiners report

Candidates were generally successful in showing that the vectors in part (c) were perpendicular. Many used the efficient approach of showing that the scalar product equaled zero, while others worked a little harder than necessary and used the cosine rule to find the angle between the two vectors.

11a. Let $u = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}$ and $w = \begin{pmatrix} 3 \\ -1 \\ p \end{pmatrix}$. Given that $u$ is perpendicular to $w$, find the value of $p$.

[3 marks]

Markscheme

evidence of equating scalar product to 0  \((M1)\)

\[2 \times 3 + 3 \times (-1) + (-1) \times p = 0\]

\[6 - 3 - p = 0, 3 - p = 0\]  \(A1\)

$p = 3$  \(A1\) \(N2\)

[3 marks]

Examiners report

Most candidates knew to set the scalar product equal to zero.
11b. Let \( v = \begin{pmatrix} 1 \\ q \\ 5 \end{pmatrix} \). Given that \(|v| = \sqrt{42}\), find the possible values of \( q \).

**Markscheme**

<table>
<thead>
<tr>
<th>Evidence of substituting into magnitude formula</th>
<th>((M1))</th>
</tr>
</thead>
<tbody>
<tr>
<td>e.g. ( \sqrt{1+q^2+25} ) ( \rightarrow ) ( 1+q^2+25 )</td>
<td>( A1 )</td>
</tr>
<tr>
<td>setting up a correct equation</td>
<td>( A1 )</td>
</tr>
<tr>
<td>e.g. ( \sqrt{1+q^2+25} = \sqrt{42} ), ( 1+q^2+25 = 42 ), ( q^2 = 16 )</td>
<td>( N2 )</td>
</tr>
</tbody>
</table>

12a. Consider the points \( P(2, -1, 5) \) and \( Q(3, -3, 8) \). Let \( L_1 \) be the line through \( P \) and \( Q \).

**Markscheme**

<table>
<thead>
<tr>
<th>Evidence of correct approach</th>
<th>( AI )</th>
</tr>
</thead>
<tbody>
<tr>
<td>e.g. ( \overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP} ), ( \begin{pmatrix} 3 \ -3 \ 8 \end{pmatrix} - \begin{pmatrix} 2 \ -1 \ 5 \end{pmatrix} )</td>
<td>( AG )</td>
</tr>
<tr>
<td>( \overrightarrow{PQ} = \begin{pmatrix} 1 \ -2 \ 3 \end{pmatrix} )</td>
<td>( N0 )</td>
</tr>
</tbody>
</table>

12b. The line \( L_1 \) may be represented by \( \begin{pmatrix} 3 \\ -3 \\ 8 \end{pmatrix} + s \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} \).

(i) What information does the vector \( \begin{pmatrix} 3 \\ -3 \\ 8 \end{pmatrix} \) give about \( L_1 \) ?

(ii) Write down another vector representation for \( L_1 \) using \( \begin{pmatrix} 3 \\ -3 \\ 8 \end{pmatrix} \).
**Markscheme**
(i) correct description \(R1\) \(N1\)

e.g. reference to \(\begin{pmatrix} 3 \\ -3 \\ 8 \end{pmatrix}\) being the position vector of a point on the line, a vector to the line, a point on the line.

(ii) any correct expression in the form \(r = a + tb\) \(A2\) \(N2\)

where \(a\) is \(\begin{pmatrix} 3 \\ -3 \\ 8 \end{pmatrix}\), and \(b\) is a scalar multiple of \(\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}\)

e.g. \(r = \begin{pmatrix} 3 \\ -3 \\ 8 \end{pmatrix} + t \begin{pmatrix} -1 \\ 2 \\ -3 \end{pmatrix}\), \(r = \begin{pmatrix} 3 + 2s \\ 3 - 4s \\ 8 + 6s \end{pmatrix}\)

\([3\text{ marks}]\)

**Examiners report**
For part (b), a number of candidates stated that the vector was a "starting point," which misses the idea that it is a position vector to some point on the line.

**12c.** The point \(T(-1, 5, p)\) lies on \(L_1\).

Find the value of \(p\).

\([3\text{ marks}]\)

**Markscheme**
\(\text{one correct equation} \ (A1)\)

e.g. \(3 + s = -1\), \(-3 - 2s = 5\)

\(s = -4 \ A1\)

\(p = -4 \ A1 \ N2\)

\([3\text{ marks}]\)

**Examiners report**
Parts (c) and (d) proved accessible to many.

**12d.** The point \(T\) also lies on \(L_2\) with equation \(\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -3 \\ 9 \\ 2 \end{pmatrix} + t \begin{pmatrix} 1 \\ -2 \\ q \end{pmatrix}\).

Show that \(q = -3\).

**Markscheme**
\(\text{one correct equation} \ (A1)\)

e.g. \(-3 + t = -1\), \(9 - 2t = 5\)

\(t = 2 \ A1\)

substituting \(t = 2\)

e.g. \(2 + 2q = -4\), \(2q = -6 \ A1\)

\(q = -3 \ AG \ N0\)

\([3\text{ marks}]\)
12e. Let $\theta$ be the obtuse angle between $L_1$ and $L_2$. Calculate the size of $\theta$.

**Markscheme**

choosing correct direction vectors $\left(\begin{array}{c} 1 \\ -2 \\ 3 \end{array}\right)$ and $\left(\begin{array}{c} 1 \\ -2 \\ -3 \end{array}\right)$ (AI)(AI)

finding correct scalar product and magnitudes (AI)(AI)(AI)

scalar product $\left(\begin{array}{c} 1 \\ -2 \\ 3 \end{array}\right)(\begin{array}{c} 1 \\ -2 \\ -3 \end{array}) = -4$

magnitudes $\sqrt{1^2 + (-2)^2 + 3^2} = \sqrt{14}$, $\sqrt{1^2 + (-2)^2 + (-3)^2} = \sqrt{14}$

evidence of substituting into scalar product $M1$

cos $\theta = \frac{-4}{3.741\ldots\times3.741\ldots}$

$\theta = 1.86$ radians (or $107^\circ$) $AI$ $N4$

[7 marks]

**Examiners report**

For part (e), a surprising number of candidates chose incorrect vectors. Few candidates seemed to have a good conceptual understanding of the vector equation of a line.

---

The vertices of the triangle PQR are defined by the position vectors

$\overrightarrow{OP} = \left(\begin{array}{c} 4 \\ -3 \\ 1 \end{array}\right)$, $\overrightarrow{OQ} = \left(\begin{array}{c} 3 \\ -1 \\ 2 \end{array}\right)$ and $\overrightarrow{OR} = \left(\begin{array}{c} 6 \\ -1 \\ 5 \end{array}\right)$.

13a. Find

(i) $\overrightarrow{PQ}$;

(ii) $\overrightarrow{PR}$.

**Markscheme**

(i) evidence of approach $MI$

e.g. $\overrightarrow{PQ} = \overrightarrow{PO} + \overrightarrow{OQ}$, $Q - P$

$\overrightarrow{PQ} = \left(\begin{array}{c} -1 \\ 2 \\ 1 \end{array}\right)$ $AI$ $N2$

(ii) $\overrightarrow{PR} = \left(\begin{array}{c} 2 \\ 2 \\ 4 \end{array}\right)$ $AI$ $N1$

[3 marks]

**Examiners report**

Combining the vectors in (a) was generally well done, although some candidates reversed the subtraction, while others calculated the magnitudes.
13b. Show that \( \cos \hat{P} \hat{Q} = \frac{1}{2} \).

**Markscheme**

**METHOD 1**

choosing correct vectors \( \overrightarrow{PQ} \) and \( \overrightarrow{PR} \) \( (A1)(A1) \)

finding \( \overrightarrow{PQ} \cdot \overrightarrow{PR} \), \( |\overrightarrow{PQ}| \), \( |\overrightarrow{PR}| \) \( (A1)(A1)(A1) \)

\[
\overrightarrow{PQ} \cdot \overrightarrow{PR} = -2 + 4 + 4 (= 6)
\]

\[
|\overrightarrow{PQ}| = \sqrt{(-1)^2 + 2^2 + 1^2} (= \sqrt{6}) , \quad |\overrightarrow{PR}| = \sqrt{2^2 + 2^2 + 4^2} (= \sqrt{24})
\]

substituting into formula for angle between two vectors \( M1 \)

e.g. \( \cos \hat{R} \hat{Q} = \frac{6}{\sqrt{6} \times \sqrt{24}} \)

simplifying to expression clearly leading to \( \frac{1}{2} \) \( A1 \)

e.g. \( \frac{6}{\sqrt{6} \times \sqrt{24}} \cdot \frac{6}{\sqrt{144}} \cdot \frac{6}{12} \)

\( \cos \hat{R} \hat{Q} = \frac{1}{2} \) \( AG \) \( N0 \)

**METHOD 2**

evidence of choosing cosine rule (seen anywhere) \( (M1) \)

\[
\overrightarrow{QR} = \begin{pmatrix} 3 \\ 0 \\ 3 \end{pmatrix} \quad (A1)
\]

\[
|\overrightarrow{QR}| = \sqrt{18}, \quad |\overrightarrow{PQ}| = \sqrt{6} \quad \text{and} \quad |\overrightarrow{PR}| = \sqrt{24} \quad (A1)(A1)(A1)
\]

\[
\cos \hat{R} \hat{Q} = \frac{(\sqrt{6})^2 + (\sqrt{24})^2 - (\sqrt{18})^2}{2 \sqrt{6} \times \sqrt{24}} \quad (A1)
\]

\[
\cos \hat{R} \hat{Q} = \frac{6 + 24 - 18}{24} \quad (= \frac{12}{24}) \quad (A1)
\]

\( \cos \hat{R} \hat{Q} = \frac{1}{2} \) \( AG \) \( N0 \)

[7 marks]

**Examiners report**

Many candidates successfully used scalar product and magnitude calculations to complete part (b). Alternatively, some used the cosine rule, and often achieved correct results. Some assumed the triangle was a right-angled triangle and thus did not earn full marks. Although PQR is indeed right-angled, in a “show that” question this attribute must be directly established.

13c. (i) Find \( \sin \hat{R} \hat{Q} \).

(ii) Hence, find the area of triangle PQR, giving your answer in the form \( a\sqrt{3} \).
**Markscheme**

(i) **METHOD 1**

evidence of appropriate approach \((M1)\)

e.g. using \(\sin^2 \hat{R}PQ + \cos^2 \hat{R}PQ = 1\) , diagram

substituting correctly \((A1)\)

e.g. \(\sin \hat{R}PQ = \sqrt{1 - \left(\frac{1}{2}\right)^2}\)

\(\sin \hat{R}PQ = \sqrt{\frac{3}{4}}\) \(A1\) \(N3\)

**METHOD 2**

since \(\cos \hat{P} = \frac{1}{2}\) , \(\hat{P} = 60^\circ\) \((A1)\)

evidence of approach

e.g. drawing a right triangle, finding the missing side \((A1)\)

\(\sin \hat{P} = \frac{\sqrt{3}}{2}\) \(A1\) \(N3\)

(ii) evidence of appropriate approach \((M1)\)

e.g. attempt to substitute into \(\frac{1}{2}ab\sin C\)

correct substitution

e.g. area = \(\frac{1}{2}\sqrt{6} \times \sqrt{24} \times \frac{\sqrt{3}}{2}\) \(A1\)

area = \(3\sqrt{3}\) \(A1\) \(N2\)

[6 marks]

**Examiners report**

Many candidates attained the value for sine in (c) with little difficulty, some using the Pythagorean identity, while others knew the side relationships in a 30-60-90 triangle. Unfortunately, a good number of candidates then used the side values of 1, 2, \(\sqrt{3}\) to find the area of PQR, instead of the magnitudes of the vectors found in (a). Furthermore, the "hence" command was sometimes neglected as the value of sine was expected to be used in the approach.

14. Two lines with equations \(r_1 = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} + s \begin{pmatrix} 5 \\ -3 \\ 2 \end{pmatrix}\) and \(r_2 = \begin{pmatrix} 0 \\ 2 \\ 2 \end{pmatrix} + t \begin{pmatrix} -3 \\ 5 \\ -1 \end{pmatrix}\) intersect at the point P. Find the coordinates of P.

**Markscheme**

evidence of appropriate approach \((M1)\)

e.g. \(\begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} + s \begin{pmatrix} 5 \\ -3 \\ 2 \end{pmatrix} = \begin{pmatrix} 9 \\ 2 \\ 2 \end{pmatrix} + t \begin{pmatrix} -3 \\ 5 \\ -1 \end{pmatrix}\)

two correct equations \(A1A1\)

e.g. \(2 + 5s = 9 - 3t\) , \(3 - 3s = 2 + 5t\) , \(-1 + 2s = 2 - t\)

attempting to solve the equations \((M1)\)

one correct parameter \(s = 2\) , \(t = -1\) \(A1\)

P is \((12, -3, 3)\) \((\text{accept} \begin{pmatrix} 12 \\ -3 \\ 3 \end{pmatrix})\) \(A1\) \(N3\)

[6 marks]
Examiners report
If this topic had been taught well then the candidates scored highly. The question was either well answered or not at all. Many candidates did not understand what was needed and tried to find the length of vectors or mid-points of lines. The other most common mistake was to use the values of the parameters to write the coordinates as \( P(2, -1). \)

15. Find the cosine of the angle between the two vectors \( 3i + 4j + 5k \) and \( 4i - 5j - 3k \). 

Markscheme

scalar product \(= 12 - 20 - 15 = -23\)
magnitudes \(= \sqrt{3^2 + 4^2 + 5^2}, \sqrt{4^2 + (-5)^2 + (-3)^2}, (\sqrt{50}, \sqrt{50})\)
substitution into formula \(M1\)
e.g. \(\cos \theta = \frac{12 - 20 - 15}{\sqrt{3^2 + 4^2 + 5^2} \cdot \sqrt{4^2 + (-5)^2 + (-3)^2}}\)
\(\cos \theta = -\frac{23}{50} = -0.46\) \(A2 \quad N4\)

[6 marks]

Examiners report

Many candidates performed well in finding the magnitudes and scalar product to use the formula for angle between vectors. Some experienced trouble with the arithmetic to obtain the required result. A significant number of candidates isolated the theta answering with \(\arccos \left( -\frac{23}{50} \right)\).

The line \(L_1\) is parallel to the \(z\)-axis. The point \(P\) has position vector \(\begin{pmatrix} 8 \\ 1 \\ 0 \end{pmatrix}\) and lies on \(L_1\).

16a. Write down the equation of \(L_1\) in the form \(r = a + tb\). 

Markscheme

\(L_1 : r = \begin{pmatrix} 8 \\ 1 \\ 0 \end{pmatrix} + t \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad A2 \quad N2\)

[2 marks]

Examiners report

Very few candidates gave a correct direction vector parallel to the \(z\)-axis. Provided they wrote down an equation here they were able to earn most of subsequent marks on follow through.

16b. The line \(L_2\) has equation \(r = \begin{pmatrix} 2 \\ 4 \\ -1 \end{pmatrix} + s \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix}\). The point \(A\) has position vector \(\begin{pmatrix} 6 \\ 2 \\ 9 \end{pmatrix}\).

Show that \(A\) lies on \(L_2\) .
Markscheme

evidence of equating $r$ and $\overrightarrow{OA}$ \((M1)\)
e.g. \[
\begin{pmatrix} 6 \\ 2 \\ 9 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \\ -1 \end{pmatrix} + s \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix}, \quad A = r
\]
one correct equation \(A1\)
e.g. $6 = 2 + 2s, 2 = 4 - s, 9 = -1 + 5s$
s = 2 \(A1\)
evidence of confirming for other two equations \(A1\)
e.g. $6 = 2 + 4, 2 = 4 - 2, 9 = -1 + 10$
so $A$ lies on $L_2$ \(AG\) \(N0\)

[4 marks]

Examiners report

For (b), many found the correct parameter but neglected to confirm it in the other two equations.

\[16c. \] Let $B$ be the point of intersection of lines $L_1$ and $L_2$. \(\text{[7 marks]}\)

(i) Show that $\overrightarrow{OB} = \begin{pmatrix} 8 \\ 1 \\ 14 \end{pmatrix}$.

(ii) Find $\overrightarrow{AB}$.

Markscheme

(i) evidence of approach \(M1\)
e.g. \[
\begin{pmatrix} 2 \\ 4 \\ -1 \end{pmatrix} + s \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix} = \begin{pmatrix} 8 \\ 1 \\ 0 \end{pmatrix} + t \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \quad L_1 = L_2
\]
one correct equation \(A1\)
e.g. $2 + 2s = 8, 4 - s = 1, -1 + 5s = t$
attempt to solve \((M1)\)
finding $s = 3$ \(A1\)
substituting \(M1\)
e.g. $\overrightarrow{OB} = \begin{pmatrix} 2 \\ 4 \\ -1 \end{pmatrix} + 3 \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix}$

\[\overrightarrow{OB} = \begin{pmatrix} 8 \\ 1 \\ 14 \end{pmatrix}, \quad AG\] \(N0\)

(ii) evidence of appropriate approach \((M1)\)
e.g. $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}, \overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

\[\overrightarrow{AB} = \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix} \quad A1 \quad N2\]

[7 marks]
**Examiners report**

In (c) some performed a trial and error approach to obtaining an integer parameter and thus did not "show" the mathematical origin of the result. Finding vector $\overrightarrow{AB}$ proved accessible.

16d.

The point C is at $(2, 1, -4)$. Let D be the point such that ABCD is a parallelogram. Find $\overrightarrow{OD}$.

**Markscheme**

- evidence of appropriate approach \((M1)\)
- e.g. $\overrightarrow{AB} = \overrightarrow{DC}$
- correct values \(A1\)
- e.g. $\overrightarrow{OD} = \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix} + \begin{pmatrix} 2 \\ 1 \\ -4 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \\ 1 \end{pmatrix}$
- \(\overrightarrow{OD} = \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix} A1 N2\)

**Examiners report**

A good number of candidates had an appropriate approach to (d), although surprisingly many subtracted $\overrightarrow{OC}$ from $\overrightarrow{AB}$ in finding $\overrightarrow{OD}$.

17.

Let $v = \begin{pmatrix} 2 \\ -3 \\ 6 \end{pmatrix}$ and $w = \begin{pmatrix} k \\ -2 \\ 4 \end{pmatrix}$, for $k > 0$. The angle between $v$ and $w$ is $\frac{\pi}{3}$.

Find the value of $k$.

**Markscheme**

- correct substitutions for $v \cdot w$ ; $|v|$ ; $|w|$ \((A1)(A1)(A1)\)
- e.g. $2k + (-3) \times (-2) + 6 \times 4$ ; $2k + 30$ ; $\sqrt{2^2 + (-3)^2 + 6^2}$ ; $\sqrt{49}$ ; $\sqrt{k^2 + (-2)^2 + 4^2}$ ; $\sqrt{k^2 + 20}$
- evidence of substituting into the formula for scalar product \((M1)\)
- e.g. $\frac{2k + 30}{\sqrt{2^2 + 20}}$
- correct substitution \(A1\)
- e.g. $\cos \frac{\pi}{3} = -\frac{2k + 30}{7\sqrt{k^2 + 20}}$
- $k = 18.8 A2 N5$

**Examiners report**

For the most part, this question was well done and candidates had little difficulty finding the scalar product, the appropriate magnitudes and then correctly substituting into the formula for the angle between vectors. However, few candidates were able to solve the resulting equation using their GDCs to obtain the correct answer. Problems arose when candidates attempted to solve the resulting equation analytically.
In this question, distance is in metres.

Toy airplanes fly in a straight line at a constant speed. Airplane 1 passes through a point A.

Its position, \( p \) seconds after it has passed through A, is given by

\[
\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ -4 \\ 0 \end{pmatrix} + p \begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix}.
\]

18a. [4 marks]

(i) Write down the coordinates of A.

(ii) Find the speed of the airplane in \( \text{m s}^{-1} \).

**Markscheme**

(i) (3, -4, 0) \( A1 \ N1 \)

(ii) choosing velocity vector \( \begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix} \) \( M1 \)

finding magnitude of velocity vector \( A1 \)

e.g. \( \sqrt{(-2)^2 + 3^2 + 1^2} \), \( \sqrt{4 + 9 + 1} \)

speed = 3.74 \( \sqrt{14} \) \( A1 \ N2 \)

**Examiners report**

Many candidates demonstrated a good understanding of the vector equation of a line and its application to a kinematics problem by correctly answering the first two parts of this question.

18b. [5 marks]

After seven seconds the airplane passes through a point B.

(i) Find the coordinates of B.

(ii) Find the distance the airplane has travelled during the seven seconds.
Markscheme

(i) substituting \( p = 7 \) \( \text{(M1)} \)
\[ B = (-11, 17, 7) \] \( \text{A1 N2} \)

(ii) METHOD 1

appropriate method to find \( \overrightarrow{AB} \) or \( \overrightarrow{BA} \) \( \text{(M1)} \)

\[ \begin{align*}
\overrightarrow{AB} & = \left( \begin{array}{c}
-14 \\
21 \\
7
\end{array} \right) \quad \text{or} \quad \overrightarrow{BA} = \left( \begin{array}{c}
14 \\
-21 \\
-7
\end{array} \right) \quad \text{(A1)} \\
\end{align*} \]

distance = 26.2 \( \sqrt{14} \) \( \text{A1 N3} \)

METHOD 2

evidence of applying distance is speed \( \times \) time \( \text{(M2)} \)

e.g. \( 3.74 \times 7 \)

distance = 26.2 \( \sqrt{14} \) \( \text{A1 N3} \)

METHOD 3

try to find \( AB^2 \), \( AB \) \( \text{(M1)} \)

\[ \begin{align*}
\text{e.g.} \quad (3 - (-11))^2 + (-4 - 17)^2 + (0 - 7)^2 & \quad , \quad \sqrt{(3 - (-11))^2 + (-4 - 17)^2 + (0 - 7)^2} \\
\text{AB}^2 & = 686, \text{AB} = \sqrt{686} \quad \text{(A1)} \\
\text{distance AB} & = 26.2 \sqrt{14} \quad \text{A1 N3} \\
\end{align*} \]

\[ 5 \text{ marks} \]

Examiners report

Many candidates demonstrated a good understanding of the vector equation of a line and its application to a kinematics problem by correctly answering the first two parts of this question.

Some knew that speed and distance were magnitudes of vectors but chose the wrong vectors to calculate magnitudes.

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18c. Airplane 2 passes through a point C. Its position \( q \) seconds after it passes through C is given by
\[ \begin{pmatrix}
x \\
y \\
z
\end{pmatrix} = \begin{pmatrix}
2 \\
-5 \\
8
\end{pmatrix} + q \begin{pmatrix}
-1 \\
2 \\
a
\end{pmatrix}, a \in \mathbb{R} \quad . \]

The angle between the flight paths of Airplane 1 and Airplane 2 is 40°. Find the two values of \( a \).

Markscheme

correct direction vectors \( \begin{pmatrix}
-2 \\
3 \\
1
\end{pmatrix} \) and \( \begin{pmatrix}
-1 \\
2 \\
a
\end{pmatrix} \) \( \text{(A1)(A1)} \)

\[ \begin{align*}
\left| \begin{array}{c}
-1 \\
2 \\
a
\end{array} \right| & = \sqrt{a^2 + 5} \quad , \quad \left| \begin{array}{c}
-2 \\
3 \\
1
\end{array} \right| \cdot \left| \begin{array}{c}
-1 \\
2 \\
a
\end{array} \right| = a + 8 \quad \text{(A1)(A1)} \\
\end{align*} \]

substituting \( M1 \)

e.g. \( \cos 40^\circ = \frac{a + 8}{\sqrt{a^2 + 5}} \)

\[ a = 3.21, a = -0.990 \quad \text{AIAI N3} \]

\[ 7 \text{ marks} \]
Examiners report

Very few candidates were able to get the two correct answers in (c) even if they set up the equation correctly. Much contorted algebra was seen and little evidence of using the GDC to solve the equation. Many made simple algebraic errors by combining unlike terms in working with the scalar product (often writing $8a$ rather than $8 + a$) or the magnitude (often writing $5a^2$ rather than $5 + a^2$).

19. Let $v = 3i + 4j + k$ and $w = i + 2j - 3k$. The vector $v + pw$ is perpendicular to $w$. Find the value of $p$. [7 marks]

Markscheme

$pw = pi + 2pj - 3pk$ (seen anywhere) (A1)

attempt to find $v + pw$ (M1)

e.g. $3i + 4j + k + p(i + 2j - 3k)$

collecting terms $(3 + p)i + (4 + 2p)j + (1 - 3p)k$ (A1)

attempt to find the dot product (M1)

e.g. $1(3 + p) + 2(4 + 2p) - 3(1 - 3p)$

setting their dot product equal to 0 (M1)

e.g. $1(3 + p) + 2(4 + 2p) - 3(1 - 3p) = 0$

simplifying (A1)

e.g. $3 + p + 8 + 4p - 3 + 9p = 0$, $14p + 8 = 0$

$p = -0.571 \ (-\frac{8}{17})$ (A1 N3)

[7 marks]

Examiners report

This question was very poorly done with many leaving it blank. Of those that did attempt it, most were able to find $v + pw$ but really did not know how to proceed from there. They tried many approaches, such as, finding magnitudes, using negative reciprocals, or calculating the angle between two vectors. A few had the idea that the scalar product should equal zero but had trouble trying to set it up.

The point O has coordinates $(0, 0, 0)$, point A has coordinates $(1, -2, 3)$ and point B has coordinates $(-3, 4, 2)$. [8 marks]

20a.

(i) Show that $\vec{AB} = \begin{pmatrix} -4 \\ 6 \\ -1 \end{pmatrix}$.

(ii) Find $\vec{BA}$. 

**Markscheme**

(i) evidence of approach \( M1 \)

e.g. \( \overrightarrow{AO} + \overrightarrow{OB} = \overrightarrow{AB} \)

\[
\overrightarrow{AB} = \begin{pmatrix} -4 \\ 6 \\ -1 \end{pmatrix} \quad AG \quad N0
\]

(ii) for choosing **correct** vectors, \( \overrightarrow{AO} \) with \( \overrightarrow{AB} \) or \( \overrightarrow{OA} \) with \( \overrightarrow{BA} \) \( (A1)(A1) \)

**Note:** Using \( \overrightarrow{AO} \) with \( \overrightarrow{BA} \) will lead to \( \pi - 0.799 \). If they then say \( \widehat{BA} = 0.799 \), this is a correct solution.

calculating \( \overrightarrow{AO} \cdot \overrightarrow{AB}, |\overrightarrow{AO}|, |\overrightarrow{AB}| \) \( (A1)(A1)(A1) \)

e.g. \( d_1 \cdot d_2 = (-1)(-4) + (2)(6) + (-3)(-1) = 19 \)

\(|d_1| = \sqrt{(-1)^2 + 2^2 + (-3)^2} = \sqrt{14}, \quad |d_2| = \sqrt{(-4)^2 + 6^2 + (-1)^2} = \sqrt{53} \)

evidence of using the formula to find the angle \( M1 \)

e.g. \( \cos \theta = \frac{(-1)(-4) + (2)(6) + (-3)(-1)}{\sqrt{(-1)^2 + 2^2 + (-3)^2} \sqrt{(-4)^2 + 6^2 + (-1)^2}} = \frac{19}{\sqrt{14} \sqrt{53}}, \quad 0.69751 \ldots \)

\( \widehat{BA} = 0.799 \) radians (accept \( 45.8^\circ \)) \( A1 \quad N3 \)

[8 marks]

**Examiners report**

Part (ai) was done well by most students. Most knew how to approach finding the angle in part (aii). The problems occurred when the incorrect vectors were chosen. If the vectors being used were stated, then follow through marks could be given.

---

20b. The line \( L_1 \) has equation \( \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -3 \\ 4 \\ 2 \end{pmatrix} + s \begin{pmatrix} -4 \\ 6 \\ -1 \end{pmatrix} \). Write down the coordinates of two points on \( L_1 \).

**Markscheme**

two correct answers \( A1A1 \)

e.g. \((-1, -2, 3), (-3, 4, 2), (-7, 10, 1), (-11, 16, 0) \) \( N2 \)

[2 marks]

**Examiners report**

Part (b) was well done.

---

20c. The line \( L_2 \) passes through \( A \) and is parallel to \( \overrightarrow{OB} \).

(i) Find a vector equation for \( L_2 \), giving your answer in the form \( r = a + tb \).

(ii) Point \( C(k, -k, 5) \) is on \( L_2 \). Find the coordinates of \( C \).
Markscheme

(i) \[ r = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} + t \begin{pmatrix} -3 \\ 4 \\ 2 \end{pmatrix} \] \hspace{1cm} A2 \hspace{1cm} N2

(ii) C on \( L_2 \), so \( \begin{pmatrix} k \\ -k \\ 5 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} + t \begin{pmatrix} -3 \\ 4 \\ 2 \end{pmatrix} \) \( (M1) \)

evidence of equating components \( (A1) \)
e.g. \( 1 - 3t = k \), \(-2 + 4t = -k \), \(5 = 3 + 2t \)
one correct value \( t = 1 \), \( k = -2 \) (seen anywhere) \( (A1) \)
coordinates of C are \((-2, 2, 5)\) \hspace{1cm} A1 \hspace{1cm} N3

[6 marks]

Examiners report

In part (ci), the error that occurred most often was the incorrect choice for the direction vector.

20d.

The line \( L_3 \) has equation \[ \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ -8 \\ 0 \end{pmatrix} + p \begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix} \]
and passes through the point C.

Find the value of \( p \) at C.

Markscheme

for setting up one (or more) correct equation using \[ \begin{pmatrix} -2 \\ 2 \\ 5 \end{pmatrix} = \begin{pmatrix} 3 \\ -8 \\ 0 \end{pmatrix} + p \begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix} \] \( (M1) \)
e.g. \( 3 + p = -2 \), \(-8 - 2p = 2 \), \(-p = 5 \)
\( p = -5 \) \hspace{1cm} A1 \hspace{1cm} N2

[2 marks]

Examiners report

Those that were able to find the coordinates in part (cii) were also able to be successful in part (d).

Consider the points A \((1, 5, 4)\), B \((3, 1, 2)\) and D \((3, k, 2)\), with \( (AD) \) perpendicular to \( (AB) \).

21a.

\begin{align*}
\text{i) } \overrightarrow{AB} : \\
\text{ii) } \overrightarrow{AD} \text{ giving your answer in terms of } k .
\end{align*}

[3 marks]
21b. Show that \( k = 7 \).

[3 marks]

Markscheme

evidence of using perpendicularity \( \Rightarrow \) scalar product = 0 \( \text{(MI)} \)

\[
\begin{align*}
\begin{pmatrix} 2 \\ -4 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ k - 5 \\ -2 \end{pmatrix} &= 0 \\
4 - 4(k - 5) + 4 &= 0 \quad A1 \\
-4k + 28 &= 0 \quad \text{(accept any correct equation clearly leading to } k = 7) \quad A1 \\
k &= 7 \quad AG \quad N0
\end{align*}
\]

[3 marks]

Examiners report

The majority of candidates correctly used the scalar product to show \( k = 7 \).

21c. The point C is such that \( \overrightarrow{BC} = \frac{1}{2} \overrightarrow{AD} \).

Find the position vector of \( C \).

[4 marks]
Examiners report
Some confusion arose in substituting $k = 7$ into $\vec{AD}$, but otherwise part (c) was well done, though finding the position vector of $C$ presented greater difficulty.
**Markscheme**

evidence of equating vectors \( (M1) \)
e.g. \( L_1 = L_2 \)
for any two correct equations \( A1A1 \)
e.g. \( 2 + s = 3 - t \), \( 5 + 2s = -3 + 3t \), \( 3 + 3s = 8 - 4t \)

attempts to solve the equations \( (M1) \)
finding one correct parameter \( (2 = -1, \ t = 2) \) \( A1 \)
the coordinates of \( T \) are \( (1, 3, 0) \) \( A1 \ \ N3 \)

[6 marks]

**Examiners report**

Those candidates prepared in this topic area answered the question particularly well, often only making some calculation error when solving the resulting system of equations. Curiously, a few candidates found correct values for \( s \) and \( t \), but when substituting back into one of the vector equations, neglected to find the \( z \)-coordinate of \( T \).

In the following diagram, \( u = \overrightarrow{AB} \) and \( v = \overrightarrow{BD} \).

The midpoint of \( \overrightarrow{AD} \) is \( E \) and \( \frac{\overrightarrow{BD}}{\overrightarrow{EC}} = \frac{1}{3} \).

Express each of the following vectors in terms of \( u \) and \( v \).

23a. \( \overrightarrow{AE} \) \( [3 \text{ marks}] \)

**Markscheme**

\( \overrightarrow{AE} = \frac{1}{2} \overrightarrow{AD} \) \( A1 \)

attempt to find \( \overrightarrow{AD} \) \( M1 \)
e.g. \( \overrightarrow{AB} + \overrightarrow{BD} \), \( u + v \)

\( \overrightarrow{AE} = \frac{1}{2}(u + v) \) \( (= \frac{1}{2}u + \frac{1}{2}v) \) \( A1 \ \ N2 \)

[3 marks]

**Examiners report**

[N/A]

23b. \( \overrightarrow{EC} \) \( [4 \text{ marks}] \)
24a. Write down the line that is parallel to $L_4$.

[1 mark]

24b. Write down the position vector of the point of intersection of $L_1$ and $L_2$.

[1 mark]
Given that $L_1$ is perpendicular to $L_3$, find the value of $a$.  

[5 marks]

**Markscheme**

choosing correct direction vectors  \( A1 \)

e.g. \[
\begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 2 \\ a \end{pmatrix}
\]

recognizing that \( a \cdot b = 0 \)  \( M1 \)

correct substitution  \( A1 \)

e.g. \(-3 - 4 - a = 0\)

\( a = -7 \)  \( A1 \)  \( N3 \)

[5 marks]

**Examiners report**

[N/A]